Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.2-Cosine/86-4.2.1.2-g-sin-^p-a+b-cos-^m

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [88]. This is test number [86].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (88)	0.00 (0)
Mathematica	100.00 (88)	0.00 (0)
Maple	100.00 (88)	0.00 (0)
Fricas	64.77 (57)	35.23 (31)
Mupad	38.64 (34)	61.36 (54)
Giac	36.36 (32)	63.64 (56)
Maxima	30.68 (27)	69.32 (61)
Sympy	26.14 (23)	73.86 (65)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

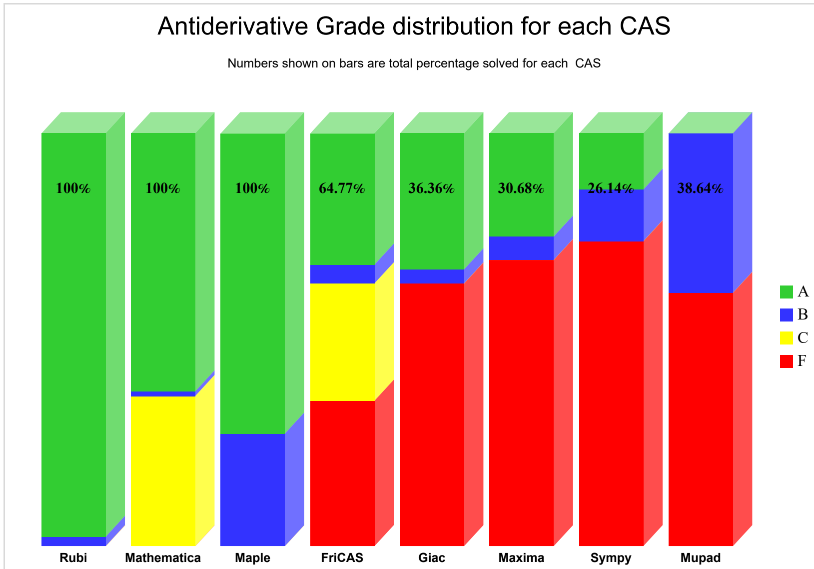
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

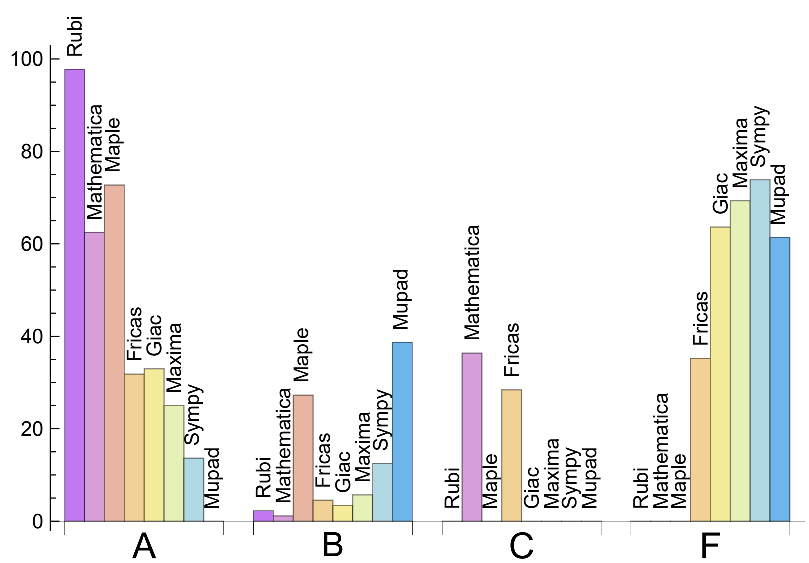
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.727	2.273	0.000	0.000
Maple	72.727	27.273	0.000	0.000
Mathematica	62.500	1.136	36.364	0.000
Giac	32.955	3.409	0.000	63.636
Fricas	31.818	4.545	28.409	35.227
Maxima	25.000	5.682	0.000	69.318
Sympy	13.636	12.500	0.000	73.864
Mupad	0.000	38.636	0.000	61.364

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	31	9.68	90.32	0.00
Mupad	54	0.00	100.00	0.00
Giac	56	100.00	0.00	0.00
Maxima	61	72.13	19.67	8.20
Sympy	65	47.69	52.31	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.19
Maxima	0.26
Giac	0.27
Rubi	1.08
Mathematica	4.84
Maple	9.89
Sympy	10.45
Mupad	10.79

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	27.85	1.32	14.00	1.00
Giac	40.88	1.22	14.00	1.01
Mupad	80.32	1.45	13.00	0.93
Fricas	107.60	1.41	105.00	1.19
Sympy	149.30	4.82	15.00	2.00
Rubi	221.59	1.06	142.50	1.00
Mathematica	403.90	1.34	101.50	1.07
Maple	672.16	2.01	227.50	1.59

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

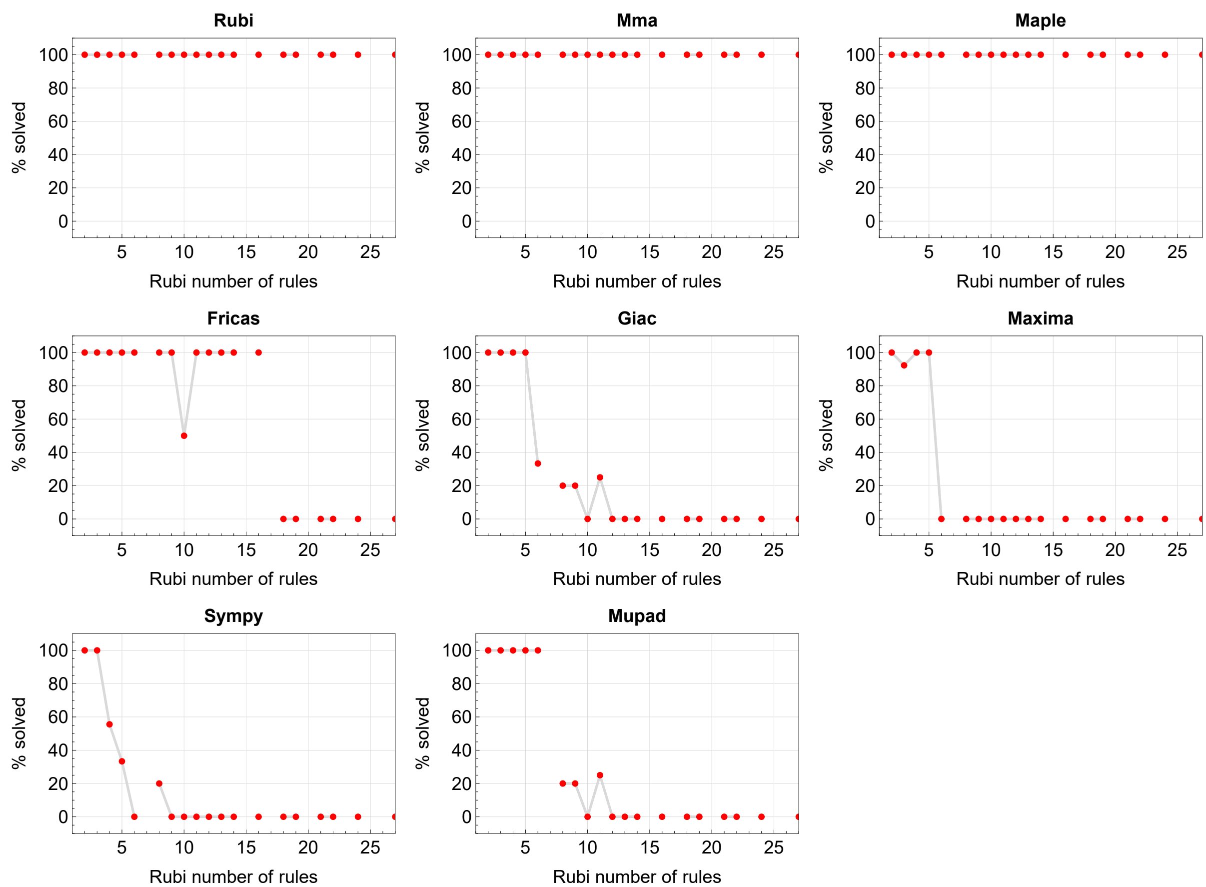


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

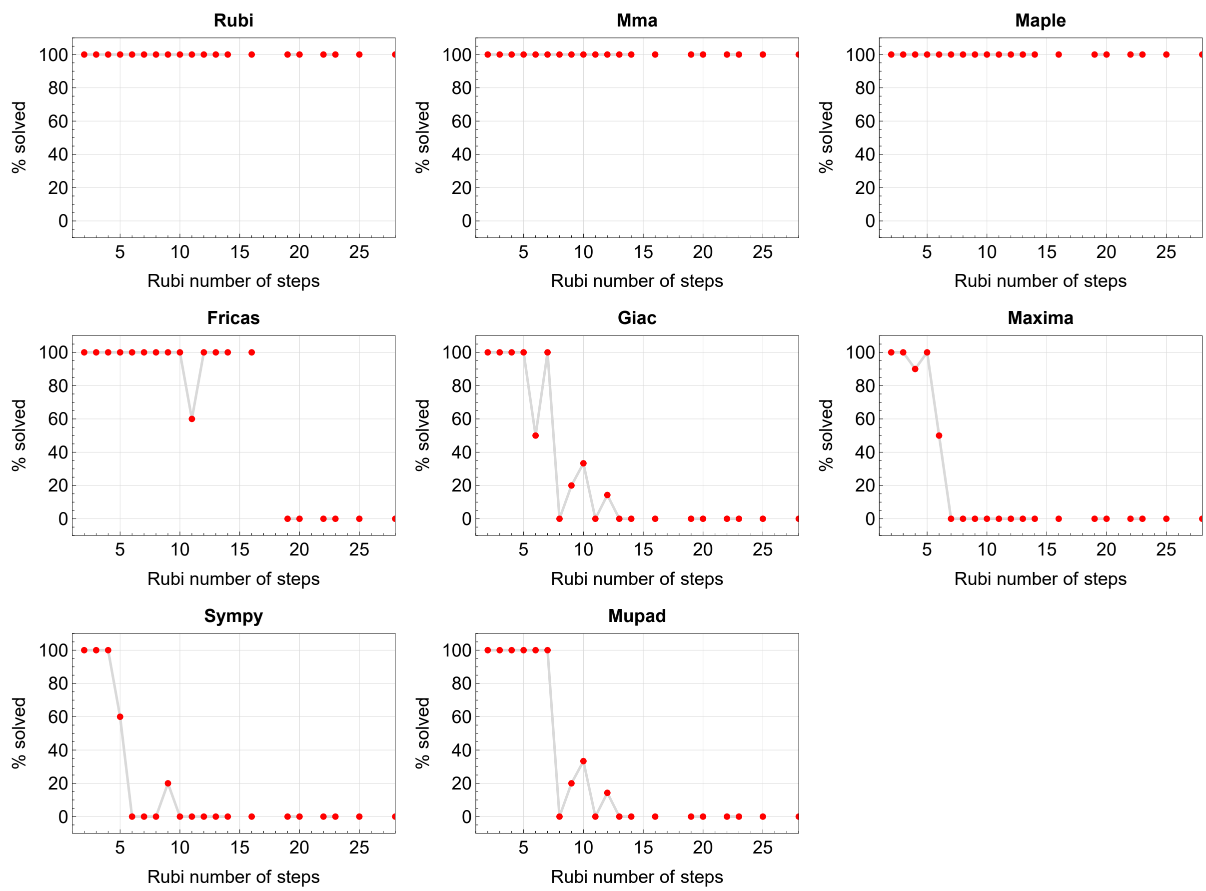


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

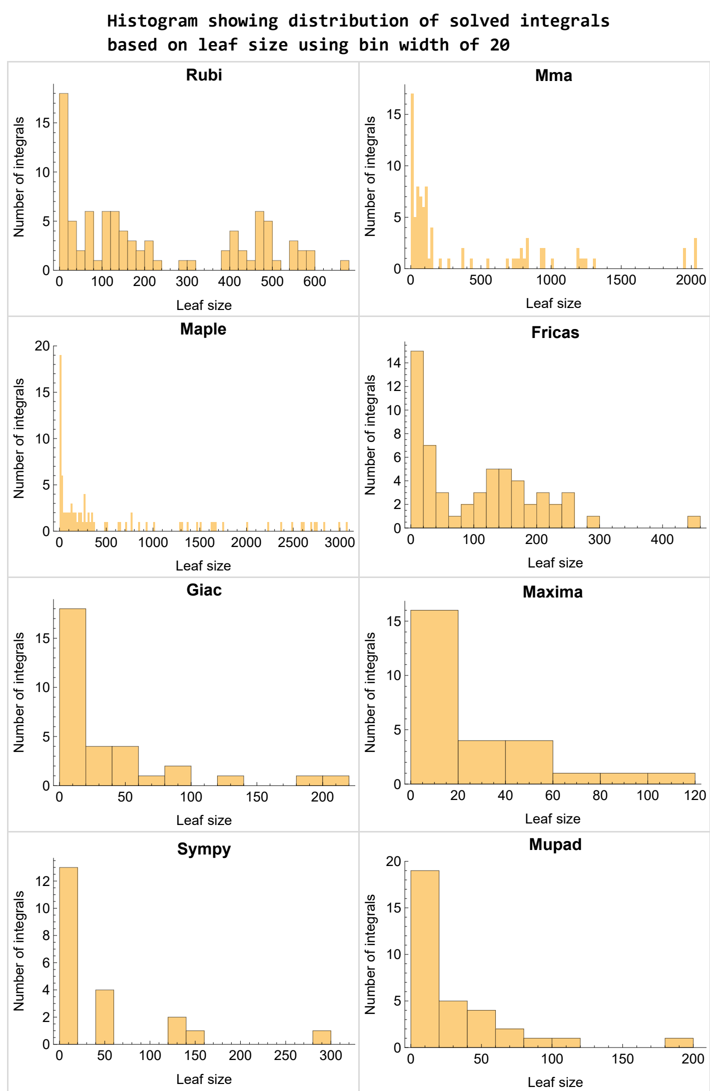


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

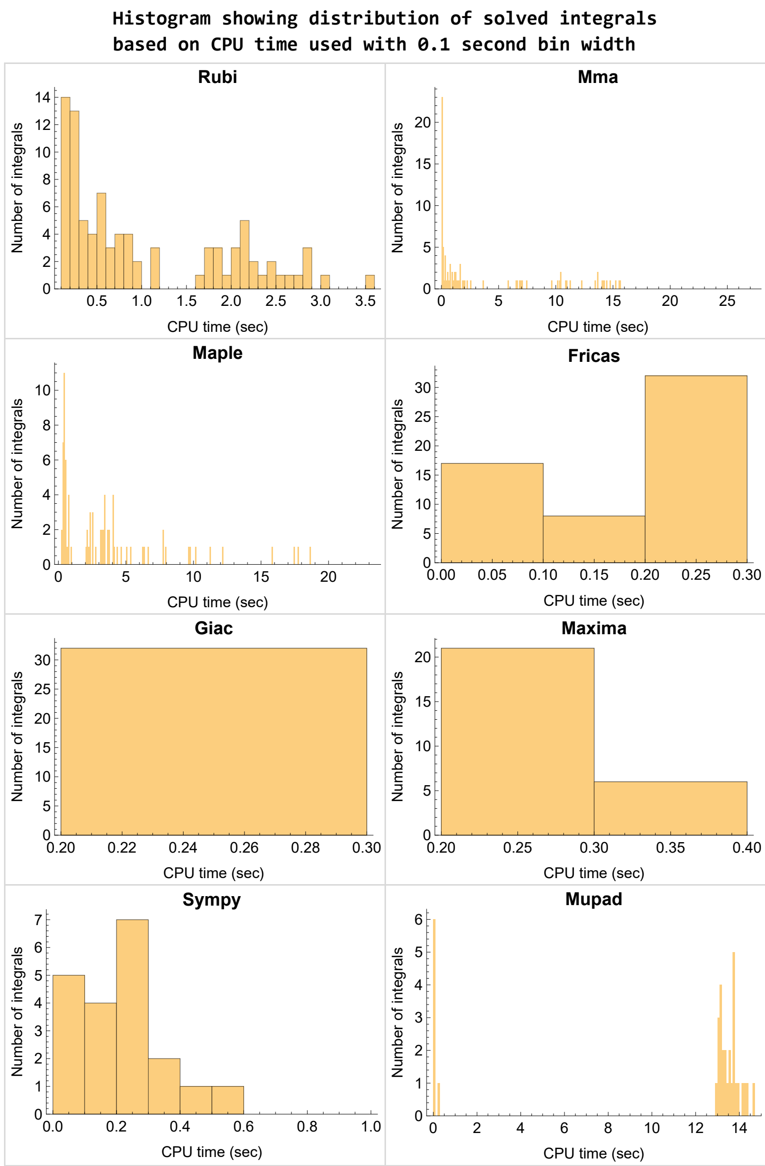


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

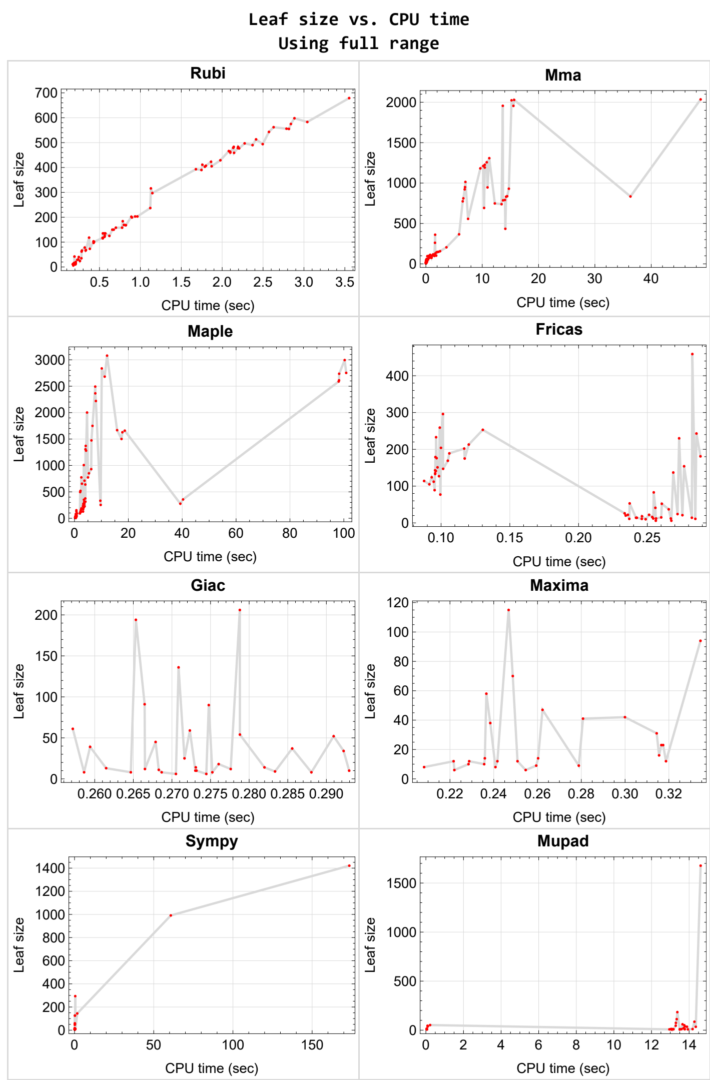


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88\}$

Mathematica $\{58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88\}$

Maple $\{58, 68, 77, 78, 79, 81, 86, 87, 88\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

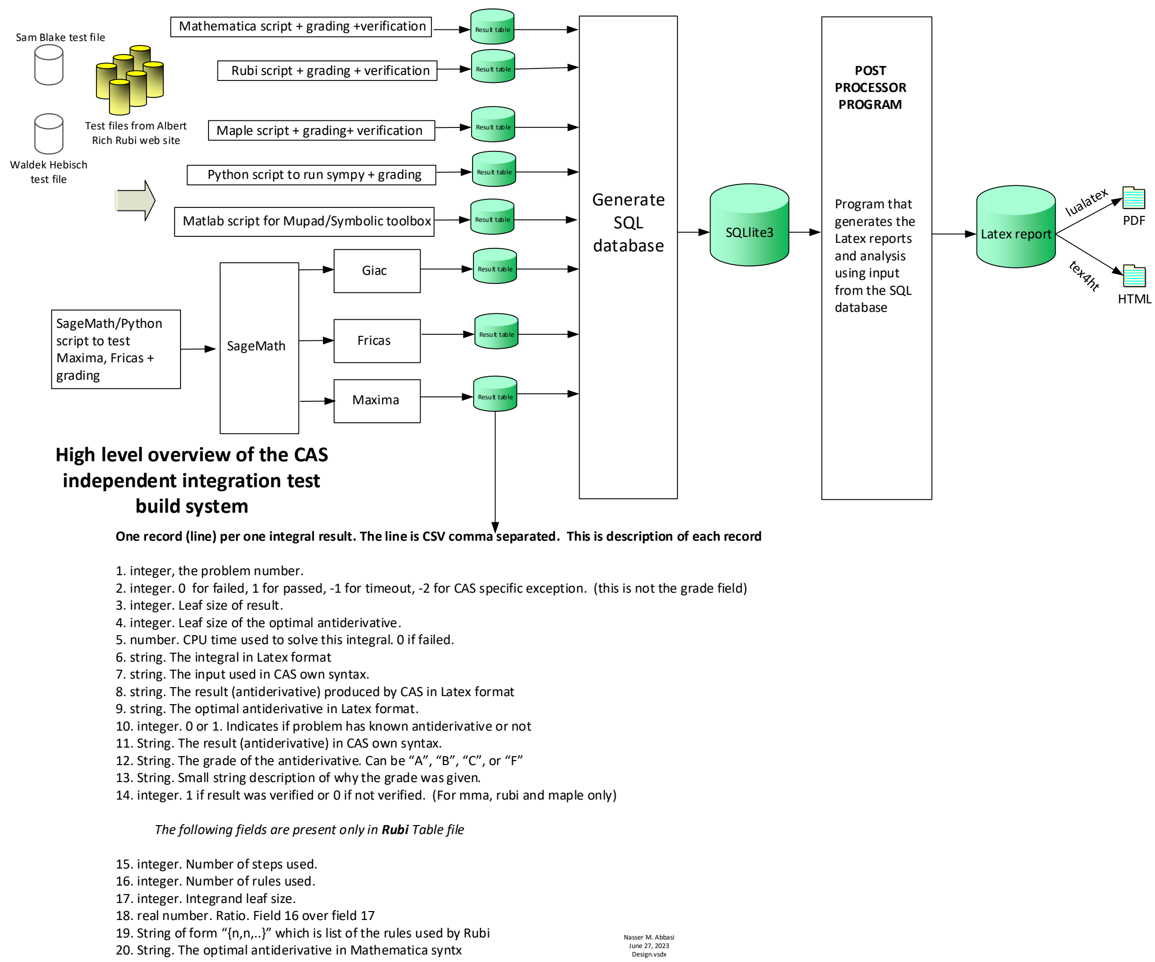
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade { 10, 11 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade { 11 }

C grade { 15, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67 }

B grade { 11, 44, 46, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 }

B grade { 10, 11, 31, 32 }

C grade { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

F normal fail { 63, 66, 74 }

F(-1) timedout fail { 58, 59, 60, 61, 62, 64, 65, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 2, 4, 5, 6, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 29, 31 }

B grade { 1, 3, 7, 9, 11 }

C grade { }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 81, 82, 83, 84 }

F(-1) timedout fail { 67, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88 }

F(-2) exception fail { 24, 26, 28, 30, 32 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31 }

B grade { 11, 24, 32 }

C grade { }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 4, 5, 10, 12, 13, 14, 15, 18, 19, 20, 21, 27 }

B grade { 1, 2, 3, 11, 16, 17, 22, 23, 25, 26, 28 }

C grade { }

F normal fail { 6, 7, 8, 9, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 62, 63, 64, 65, 66, 73, 74 }

F(-1) timedout fail { 24, 33, 40, 41, 42, 48, 49, 50, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	30	25	24	94	24	294	45	34
N.S.	1	0.97	0.81	0.77	3.03	0.77	9.48	1.45	1.10
time (sec)	N/A	0.243	0.200	0.428	0.334	0.272	0.440	0.268	14.354

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	16	13	16	14	14	51	14	11
N.S.	1	0.84	0.68	0.84	0.74	0.74	2.68	0.74	0.58
time (sec)	N/A	0.202	0.015	0.445	0.236	0.282	0.248	0.282	14.184

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	11	42	10	46	25	10
N.S.	1	1.00	1.31	0.85	3.23	0.77	3.54	1.92	0.77
time (sec)	N/A	0.202	0.044	0.405	0.300	0.249	0.177	0.272	13.503

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	12	12	13	12	12	8	10	10
N.S.	1	1.20	1.20	1.30	1.20	1.20	0.80	1.00	1.00
time (sec)	N/A	0.193	0.008	0.348	0.242	0.267	0.063	0.293	13.597

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	12	11	5	8	8
N.S.	1	1.00	0.91	0.82	1.09	1.00	0.45	0.73	0.73
time (sec)	N/A	0.172	0.007	0.204	0.251	0.237	0.098	0.288	13.936

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	29	42	20	31	37	0	34	20
N.S.	1	1.26	1.83	0.87	1.35	1.61	0.00	1.48	0.87
time (sec)	N/A	0.233	0.032	0.562	0.314	0.266	0.000	0.292	13.705

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	25	41	26	0	37	35
N.S.	1	1.00	1.25	1.04	1.71	1.08	0.00	1.54	1.46
time (sec)	N/A	0.251	0.192	0.592	0.281	0.234	0.000	0.286	13.884

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	62	60	36	58	83	0	52	45
N.S.	1	1.27	1.22	0.73	1.18	1.69	0.00	1.06	0.92
time (sec)	N/A	0.264	0.126	0.630	0.237	0.255	0.000	0.291	0.088

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	38	45	70	53	0	59	45
N.S.	1	0.95	1.03	1.22	1.89	1.43	0.00	1.59	1.22
time (sec)	N/A	0.278	0.187	0.546	0.249	0.237	0.000	0.272	13.288

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	11	5	8	9	11	10	9	7
N.S.	1	2.20	1.00	1.60	1.80	2.20	2.00	1.80	1.40
time (sec)	N/A	0.180	0.007	0.420	0.279	0.246	0.054	0.283	0.062

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	13	7	12	9	11	8	11	9
N.S.	1	4.33	2.33	4.00	3.00	3.67	2.67	3.67	3.00
time (sec)	N/A	0.182	0.009	0.422	0.259	0.285	0.054	0.268	13.169

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	12	7	6	6	5	6	6
N.S.	1	1.00	2.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.174	0.011	0.309	0.222	0.256	0.164	0.271	13.072

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	6	6	5	6	6
N.S.	1	1.00	1.20	0.90	0.60	0.60	0.50	0.60	0.60
time (sec)	N/A	0.178	0.012	0.318	0.255	0.267	0.164	0.274	13.500

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	11	23	18	7	10	10
N.S.	1	1.00	1.29	0.79	1.64	1.29	0.50	0.71	0.71
time (sec)	N/A	0.185	0.008	0.358	0.317	0.246	0.215	0.273	13.112

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	23	16	8	12	10
N.S.	1	1.00	1.62	1.06	1.44	1.00	0.50	0.75	0.62
time (sec)	N/A	0.190	0.016	0.349	0.317	0.254	0.353	0.267	13.771

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	10	12	58	10	10
N.S.	1	1.00	1.30	1.10	1.00	1.20	5.80	1.00	1.00
time (sec)	N/A	0.207	0.023	0.468	0.228	0.256	0.195	0.273	13.760

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	10	12	58	12	10
N.S.	1	1.00	1.08	0.92	0.83	1.00	4.83	1.00	0.83
time (sec)	N/A	0.205	0.019	0.559	0.236	0.254	0.202	0.278	13.121

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	14	14	8	8
N.S.	1	1.00	1.20	0.90	0.80	1.40	1.40	0.80	0.80
time (sec)	N/A	0.180	0.010	0.423	0.241	0.242	0.244	0.265	0.037

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	8	14	15	8	8
N.S.	1	1.00	1.00	0.92	0.67	1.17	1.25	0.67	0.67
time (sec)	N/A	0.180	0.013	0.426	0.208	0.242	0.242	0.259	12.952

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	12	20	7	8	8
N.S.	1	1.00	0.86	0.64	0.86	1.43	0.50	0.57	0.57
time (sec)	N/A	0.189	0.083	0.311	0.222	0.234	0.341	0.275	13.047

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	12	22	10	8	8
N.S.	1	1.00	0.75	0.56	0.75	1.38	0.62	0.50	0.50
time (sec)	N/A	0.192	0.089	0.396	0.229	0.236	0.551	0.269	13.117

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	14	21	126	14	14
N.S.	1	1.00	1.29	1.07	1.00	1.50	9.00	1.00	1.00
time (sec)	N/A	0.211	0.011	0.574	0.260	0.276	0.264	0.273	0.047

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	29	17	16	22	126	18	16
N.S.	1	1.00	1.45	0.85	0.80	1.10	6.30	0.90	0.80
time (sec)	N/A	0.206	0.015	0.474	0.316	0.251	0.256	0.276	0.047

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	125	96	152	0	243	0	194	1677
N.S.	1	1.20	0.92	1.46	0.00	2.34	0.00	1.87	16.12
time (sec)	N/A	0.623	0.263	0.712	0.000	0.286	0.000	0.265	14.612

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	38	41	1421	39	38
N.S.	1	1.00	1.00	0.98	0.95	1.02	35.52	0.98	0.95
time (sec)	N/A	0.244	0.088	0.795	0.238	0.256	173.474	0.259	0.100

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	73	54	78	0	154	991	90	74
N.S.	1	1.24	0.92	1.32	0.00	2.61	16.80	1.53	1.25
time (sec)	N/A	0.378	0.109	0.464	0.000	0.277	60.780	0.275	13.298

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	15	17	13	12
N.S.	1	1.00	1.00	1.08	1.00	1.25	1.42	1.08	1.00
time (sec)	N/A	0.184	0.025	0.421	0.319	0.260	0.093	0.261	13.042

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	137	144	61	38
N.S.	1	1.00	0.98	0.86	0.00	3.26	3.43	1.45	0.90
time (sec)	N/A	0.196	0.031	0.286	0.000	0.269	1.710	0.257	13.776

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	66	50	47	47	52	0	54	52
N.S.	1	1.25	0.94	0.89	0.89	0.98	0.00	1.02	0.98
time (sec)	N/A	0.273	0.062	0.754	0.262	0.261	0.000	0.279	0.226

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	78	66	78	0	230	0	91	86
N.S.	1	1.16	0.99	1.16	0.00	3.43	0.00	1.36	1.28
time (sec)	N/A	0.310	0.392	0.592	0.000	0.273	0.000	0.266	14.283

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	118	99	91	115	181	0	136	112
N.S.	1	1.28	1.08	0.99	1.25	1.97	0.00	1.48	1.22
time (sec)	N/A	0.364	0.510	0.934	0.247	0.289	0.000	0.271	13.340

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	135	112	127	0	459	0	206	184
N.S.	1	1.23	1.02	1.15	0.00	4.17	0.00	1.87	1.67
time (sec)	N/A	0.567	0.763	0.729	0.000	0.283	0.000	0.279	13.374

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	135	108	127	0	143	0	0	0
N.S.	1	1.05	0.84	0.98	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.543	1.188	3.177	0.000	0.096	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	101	80	171	0	127	0	0	0
N.S.	1	1.01	0.80	1.71	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.431	0.776	3.214	0.000	0.098	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	101	80	116	0	105	0	0	0
N.S.	1	1.01	0.80	1.16	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.424	0.676	2.161	0.000	0.091	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	117	0	89	0	0	60
N.S.	1	1.00	0.88	1.72	0.00	1.31	0.00	0.00	0.88
time (sec)	N/A	0.326	0.316	2.332	0.000	0.095	0.000	0.000	13.648

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	54	92	0	77	0	0	50
N.S.	1	1.00	0.82	1.39	0.00	1.17	0.00	0.00	0.76
time (sec)	N/A	0.327	0.387	2.013	0.000	0.099	0.000	0.000	13.744

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	58	153	0	114	0	0	0
N.S.	1	1.01	0.60	1.59	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.432	0.327	2.359	0.000	0.088	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	103	59	124	0	135	0	0	0
N.S.	1	1.01	0.58	1.22	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.427	0.402	2.341	0.000	0.096	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	135	74	187	0	179	0	0	0
N.S.	1	1.03	0.56	1.43	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.541	0.568	2.568	0.000	0.096	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	184	157	252	0	202	0	0	0
N.S.	1	0.95	0.81	1.31	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.788	2.519	9.724	0.000	0.117	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	150	116	332	0	175	0	0	0
N.S.	1	0.97	0.75	2.16	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.660	1.320	9.602	0.000	0.117	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	150	117	229	0	151	0	0	0
N.S.	1	0.97	0.76	1.49	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.662	1.295	3.145	0.000	0.097	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	117	83	272	0	124	0	0	0
N.S.	1	1.03	0.73	2.39	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.526	0.703	3.290	0.000	0.093	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	115	79	170	0	112	0	0	0
N.S.	1	1.01	0.69	1.49	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.545	0.987	2.540	0.000	0.095	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	121	75	283	0	141	0	0	0
N.S.	1	1.03	0.64	2.40	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.560	0.961	3.485	0.000	0.096	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	125	76	202	0	176	0	0	0
N.S.	1	1.01	0.61	1.63	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.561	1.004	3.371	0.000	0.097	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	158	109	351	0	233	0	0	0
N.S.	1	0.96	0.66	2.13	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.705	1.413	3.495	0.000	0.096	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	237	205	276	0	253	0	0	0
N.S.	1	0.98	0.85	1.14	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	1.114	3.651	39.301	0.000	0.130	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	203	149	356	0	213	0	0	0
N.S.	1	1.00	0.74	1.76	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.949	2.243	40.314	0.000	0.120	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	203	147	272	0	189	0	0	0
N.S.	1	1.00	0.73	1.35	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.957	2.039	3.647	0.000	0.106	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	169	105	314	0	147	0	0	0
N.S.	1	1.05	0.65	1.95	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.820	1.226	4.064	0.000	0.101	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	158	98	210	0	133	0	0	0
N.S.	1	1.01	0.62	1.34	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.792	1.506	3.478	0.000	0.096	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	168	101	313	0	169	0	0	0
N.S.	1	1.02	0.61	1.90	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.818	1.100	3.637	0.000	0.105	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	168	102	226	0	204	0	0	0
N.S.	1	0.99	0.60	1.34	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.826	1.834	3.766	0.000	0.100	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	199	130	375	0	259	0	0	0
N.S.	1	1.04	0.68	1.95	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.885	1.680	4.020	0.000	0.099	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	202	144	265	0	296	0	0	0
N.S.	1	1.05	0.75	1.37	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.912	1.902	3.319	0.000	0.101	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	544	555	2035	930	0	0	0	0	0
N.S.	1	1.02	3.74	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.826	48.769	6.219	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	461	459	834	851	0	0	0	0	0
N.S.	1	1.00	1.81	1.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.211	36.314	5.372	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	474	478	1955	773	0	0	0	0	0
N.S.	1	1.01	4.12	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.224	15.539	5.013	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	399	390	692	639	0	0	0	0	0
N.S.	1	0.98	1.73	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.775	10.333	4.025	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	410	407	434	654	0	0	0	0	0
N.S.	1	0.99	1.06	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.826	14.106	2.770	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	302	297	361	496	0	0	0	0	0
N.S.	1	0.98	1.20	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.143	1.648	2.114	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	307	316	261	519	0	0	0	0	0
N.S.	1	1.03	0.85	1.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.174	1.616	2.232	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	426	405	791	774	0	0	0	0	0
N.S.	1	0.95	1.86	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.921	14.030	2.551	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	447	429	1192	711	0	0	0	0	0
N.S.	1	0.96	2.67	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.946	10.452	3.720	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	501	494	811	1007	0	0	0	0	0
N.S.	1	0.99	1.62	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.490	6.656	3.464	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	557	556	2029	1658	0	0	0	0	0
N.S.	1	1.00	3.64	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.791	15.671	18.677	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	473	460	739	1628	0	0	0	0	0
N.S.	1	0.97	1.56	3.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.129	13.435	17.785	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	487	479	1956	1501	0	0	0	0	0
N.S.	1	0.98	4.02	3.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.249	13.655	17.464	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	404	393	366	1668	0	0	0	0	0
N.S.	1	0.97	0.91	4.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.700	5.896	15.829	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	418	411	557	1370	0	0	0	0	0
N.S.	1	0.98	1.33	3.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.806	7.490	4.174	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	438	403	786	1306	0	0	0	0	0
N.S.	1	0.92	1.79	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.839	13.695	4.089	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	445	423	1182	1280	0	0	0	0	0
N.S.	1	0.95	2.66	2.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.909	9.667	4.308	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	507	490	774	2002	0	0	0	0	0
N.S.	1	0.97	1.53	3.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.451	6.541	4.672	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	530	513	1257	1474	0	0	0	0	0
N.S.	1	0.97	2.37	2.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.417	10.807	6.300	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	590	583	950	1749	0	0	0	0	0
N.S.	1	0.99	1.61	2.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.123	6.949	6.688	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	590	543	930	2995	0	0	0	0	0
N.S.	1	0.92	1.58	5.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.713	14.732	100.322	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	604	562	2024	2752	0	0	0	0	0
N.S.	1	0.93	3.35	4.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.742	15.205	100.882	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	498	466	837	2736	0	0	0	0	0
N.S.	1	0.94	1.68	5.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.136	14.435	98.270	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	512	483	946	2589	0	0	0	0	0
N.S.	1	0.94	1.85	5.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.234	10.942	98.093	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	520	464	831	2612	0	0	0	0	0
N.S.	1	0.89	1.60	5.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.146	14.238	98.230	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	534	483	1211	2493	0	0	0	0	0
N.S.	1	0.90	2.27	4.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.248	10.180	7.706	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	529	477	748	2365	0	0	0	0	0
N.S.	1	0.90	1.41	4.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.272	12.239	7.733	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	535	497	1226	2220	0	0	0	0	0
N.S.	1	0.93	2.29	4.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.371	10.454	7.953	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	611	575	922	2837	0	0	0	0	0
N.S.	1	0.94	1.51	4.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.909	6.894	10.136	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	629	598	1308	2681	0	0	0	0	0
N.S.	1	0.95	2.08	4.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.991	11.256	11.217	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	700	679	1014	3079	0	0	0	0	0
N.S.	1	0.97	1.45	4.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.654	7.019	12.100	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [88] had the largest ratio of [1.08000000000000007]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	0.97	13	0.385
2	A	4	3	0.84	13	0.231
3	A	3	3	1.00	13	0.231
4	A	4	3	1.20	11	0.273
5	A	2	2	1.00	8	0.250
6	A	5	4	1.26	11	0.364
7	A	6	5	1.00	13	0.385
8	A	5	4	1.27	13	0.308
9	A	6	5	0.95	13	0.385
10	B	4	3	2.20	13	0.231
11	B	4	3	4.33	15	0.200
12	A	4	3	1.00	9	0.333
13	A	4	3	1.00	11	0.273
14	A	3	3	1.00	11	0.273
15	A	3	3	1.00	13	0.231
16	A	5	4	1.00	11	0.364
17	A	5	4	1.00	13	0.308
18	A	4	3	1.00	9	0.333
19	A	4	3	1.00	11	0.273
20	A	2	2	1.00	11	0.182
21	A	2	2	1.00	13	0.154
22	A	5	4	1.00	11	0.364

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	5	4	1.00	13	0.308
24	A	12	11	1.20	13	0.846
25	A	5	4	1.00	13	0.308
26	A	9	8	1.24	13	0.615
27	A	4	3	1.00	11	0.273
28	A	4	3	1.00	8	0.375
29	A	5	4	1.25	11	0.364
30	A	7	6	1.16	13	0.462
31	A	5	4	1.28	13	0.308
32	A	10	9	1.23	13	0.692
33	A	10	10	1.05	23	0.435
34	A	8	8	1.01	23	0.348
35	A	8	8	1.01	23	0.348
36	A	6	6	1.00	23	0.261
37	A	6	6	1.00	23	0.261
38	A	8	8	1.01	23	0.348
39	A	8	8	1.01	23	0.348
40	A	10	10	1.03	23	0.435
41	A	13	13	0.95	25	0.520
42	A	11	11	0.97	25	0.440
43	A	11	11	0.97	25	0.440
44	A	9	9	1.03	25	0.360
45	A	9	9	1.01	25	0.360
46	A	9	9	1.03	25	0.360
47	A	9	9	1.01	25	0.360
48	A	11	11	0.96	25	0.440
49	A	16	16	0.98	25	0.640
50	A	14	14	1.00	25	0.560
51	A	14	14	1.00	25	0.560
52	A	12	12	1.05	25	0.480
53	A	12	12	1.01	25	0.480
54	A	12	12	1.02	25	0.480
55	A	12	12	0.99	25	0.480
56	A	12	12	1.04	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	12	12	1.05	25	0.480
58	A	25	24	1.02	25	0.960
59	A	22	21	1.00	25	0.840
60	A	22	21	1.01	25	0.840
61	A	19	18	0.98	25	0.720
62	A	19	18	0.99	25	0.720
63	A	11	10	0.98	25	0.400
64	A	11	10	1.03	25	0.400
65	A	19	18	0.95	25	0.720
66	A	19	18	0.96	25	0.720
67	A	22	21	0.99	25	0.840
68	A	25	24	1.00	25	0.960
69	A	22	21	0.97	25	0.840
70	A	22	21	0.98	25	0.840
71	A	19	18	0.97	25	0.720
72	A	20	19	0.98	25	0.760
73	A	19	18	0.92	25	0.720
74	A	19	18	0.95	25	0.720
75	A	22	21	0.97	25	0.840
76	A	22	21	0.97	25	0.840
77	A	25	24	0.99	25	0.960
78	A	25	24	0.92	25	0.960
79	A	25	24	0.93	25	0.960
80	A	22	21	0.94	25	0.840
81	A	22	21	0.94	25	0.840
82	A	22	21	0.89	25	0.840
83	A	23	22	0.90	25	0.880
84	A	22	21	0.90	25	0.840
85	A	22	21	0.93	25	0.840
86	A	25	24	0.94	25	0.960
87	A	25	24	0.95	25	0.960
88	A	28	27	0.97	25	1.080

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx$

3.1.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}-\frac{\sin^3(x)}{3a}$$

output
```
1/2*x/a-1/2*cos(x)*sin(x)/a-1/3*sin(x)^3/a
```

3.1.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx=\frac{6x-3\sin(x)-3\sin(2x)+\sin(3x)}{12a}$$

input
```
Integrate[Sin[x]^4/(a + a*Cos[x]),x]
```

output
```
(6*x - 3*Sin[x] - 3*Sin[2*x] + Sin[3*x])/(12*a)
```

3.1.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^4}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\int \sin^2(x)dx}{a}-\frac{\sin^3(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(x)^2dx}{a}-\frac{\sin^3(x)}{3a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)}{a}-\frac{\sin^3(x)}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)}{a}-\frac{\sin^3(x)}{3a}$$

input
```
Int[Sin[x]^4/(a + a*Cos[x]),x]
```

output
```
-1/3*Sin[x]^3/a + (x/2 - (Cos[x]*Sin[x])/2)/a
```

3.1.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

3.1.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{6x-3\sin(x)+\sin(3x)-3\sin(2x)}{12a}$	24
risch	$\frac{x}{2a}-\frac{\sin(x)}{4a}+\frac{\sin(3x)}{12a}-\frac{\sin(2x)}{4a}$	33
default	$\frac{\frac{16\left(\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{16}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{6}-\frac{\tan\left(\frac{x}{2}\right)}{16}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	48
norman	$\frac{\frac{\tan^7\left(\frac{x}{2}\right)}{a}-\frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a}-\frac{5\left(\tan^5\left(\frac{x}{2}\right)\right)}{3a}+\frac{x}{2a}-\frac{\tan\left(\frac{x}{2}\right)}{a}+\frac{2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	108

input
```
int(sin(x)^4/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*x-3*sin(x)+sin(3*x)-3*sin(2*x))/a
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = \frac{\left(2\,\cos\left(x\right)^2 - 3\,\cos\left(x\right) - 2\right)\sin\left(x\right) + 3\,x}{6\,a}$$

input
```
integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
1/6*((2*cos(x)^2 - 3*cos(x) - 2)*sin(x) + 3*x)/a
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(22) = 44.

Time = 0.44 (sec) , antiderivative size = 294, normalized size of antiderivative = 9.48

$$\begin{aligned}\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx &= \frac{3x\tan^6\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ &+ \frac{9x\tan^4\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ &+ \frac{9x\tan^2\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ &+ \frac{3x}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ &+ \frac{6\tan^5\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ &- \frac{16\tan^3\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a} \\ &- \frac{6\tan\left(\frac{x}{2}\right)}{6a\tan^6\left(\frac{x}{2}\right) + 18a\tan^4\left(\frac{x}{2}\right) + 18a\tan^2\left(\frac{x}{2}\right) + 6a}\end{aligned}$$

input
```
integrate(sin(x)**4/(a+a*cos(x)),x)
```

output
```
3*x*tan(x/2)**6/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2 + 6
*a) + 9*x*tan(x/2)**4/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)*
*2 + 6*a) + 9*x*tan(x/2)**2/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan
(x/2)**2 + 6*a) + 3*x/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)*
*2 + 6*a) + 6*tan(x/2)**5/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x
/2)**2 + 6*a) - 16*tan(x/2)**3/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*
tan(x/2)**2 + 6*a) - 6*tan(x/2)/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a
*tan(x/2)**2 + 6*a)
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(25) = 50$.

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.03

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = -\frac{\frac{3\sin(x)}{\cos(x)+1} + \frac{8\sin(x)^3}{(\cos(x)+1)^3} - \frac{3\sin(x)^5}{(\cos(x)+1)^5}}{3\left(a + \frac{3a\sin(x)^2}{(\cos(x)+1)^2} + \frac{3a\sin(x)^4}{(\cos(x)+1)^4} + \frac{a\sin(x)^6}{(\cos(x)+1)^6}\right)} + \frac{\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

input
```
integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
-1/3*(3*sin(x)/(cos(x) + 1) + 8*sin(x)^3/(cos(x) + 1)^3 - 3*sin(x)^5/(cos(
x) + 1)^5)/(a + 3*a*sin(x)^2/(cos(x) + 1)^2 + 3*a*sin(x)^4/(cos(x) + 1)^4
+ a*sin(x)^6/(cos(x) + 1)^6) + arctan(sin(x)/(cos(x) + 1))/a
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = \frac{x}{2a} + \frac{3\tan\left(\frac{1}{2}x\right)^5 - 8\tan\left(\frac{1}{2}x\right)^3 - 3\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^3 a}$$

input
```
integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

output
```
1/2*x/a + 1/3*(3*tan(1/2*x)^5 - 8*tan(1/2*x)^3 - 3*tan(1/2*x))/((tan(1/2*x
)^2 + 1)^3*a)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = \frac{x}{2\,a} - \frac{\sin(x)}{3\,a} + \frac{\cos(x)^2\,\sin(x)}{3\,a} - \frac{\cos(x)\,\sin(x)}{2\,a}$$

input
```
int(sin(x)^4/(a + a*cos(x)),x)
```

output
```
x/(2*a) - sin(x)/(3*a) + (cos(x)^2*sin(x))/(3*a) - (cos(x)*sin(x))/(2*a)
```

3.2 $\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx$

3.2.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=-\frac{\cos(x)}{a}+\frac{\cos^2(x)}{2a}$$

output
```
-cos(x)/a+1/2*cos(x)^2/a
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=\frac{2\sin^4\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sin[x]^3/(a + a*Cos[x]),x]
```

output
```
(2*Sin[x/2]^4)/a
```

3.2.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$-\frac{\int (a-a\cos(x))d(a\cos(x))}{a^3}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a-a\cos(x))^2}{2a^3}$$

input
```
Int[Sin[x]^3/(a + a*Cos[x]),x]
```

output
```
(a - a*Cos[x])^2/(2*a^3)
```

3.2.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.2.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^{2}(x)\right)}{2}-\cos(x)}{a}$	16
default	$\frac{\frac{\left(\cos^{2}(x)\right)}{2}-\cos(x)}{a}$	16
parallelrisch	$\frac{\cos(2x)-5-4\cos(x)}{4a}$	16
risch	$-\frac{\cos(x)}{a}+\frac{\cos(2x)}{4a}$	18
norman	$\frac{-\frac{2}{a}-\frac{4\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a}-\frac{6\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}$	40

input
```
int(sin(x)^3/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*cos(x)^2-cos(x))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx=\frac{\cos\left(x\right)^{2}-2\,\cos\left(x\right)}{2\,a}$$

input
```
integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
1/2*(cos(x)^2 - 2*cos(x))/a
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\,dx = -\frac{4\tan^2\left(\frac{x}{2}\right)}{a\tan^4\left(\frac{x}{2}\right) + 2a\tan^2\left(\frac{x}{2}\right) + a} - \frac{2}{a\tan^4\left(\frac{x}{2}\right) + 2a\tan^2\left(\frac{x}{2}\right) + a}$$

input
```
integrate(sin(x)**3/(a+a*cos(x)),x)
```

output
```
-4*tan(x/2)**2/(a*tan(x/2)**4 + 2*a*tan(x/2)**2 + a) - 2/(a*tan(x/2)**4 +
2*a*tan(x/2)**2 + a)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\,dx = \frac{\cos(x)^2 - 2\cos(x)}{2a}$$

input
```
integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/2*(cos(x)^2 - 2*cos(x))/a
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\,dx = \frac{\cos(x)^2 - 2\cos(x)}{2a}$$

input
```
integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="giac")
```

output
```
1/2*(cos(x)^2 - 2*cos(x))/a
```

3.2.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\sin^3(x)}{a + a\cos(x)}\,dx = \frac{\cos(x)\,(\cos(x) - 2)}{2\,a}$$

input
```
int(sin(x)^3/(a + a*cos(x)),x)
```

output
```
(cos(x)*(cos(x) - 2))/(2*a)
```

3.3 $\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx$

3.3.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx=\frac{x}{a}-\frac{\sin(x)}{a}$$

output
```
x/a-sin(x)/a
```

3.3.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx=\frac{2\left(\frac{x}{2}-\frac{\sin(x)}{2}\right)}{a}$$

input
```
Integrate[Sin[x]^2/(a + a*Cos[x]),x]
```

output
```
(2*(x/2 - Sin[x]/2))/a
```

3.3.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\int 1dx}{a}-\frac{\sin(x)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{a}-\frac{\sin(x)}{a}$$

input
```
Int[Sin[x]^2/(a + a*Cos[x]),x]
```

output
```
x/a - Sin[x]/a
```

3.3.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

3.3.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{x-\sin(x)}{a}$	11
risch	$\frac{x}{a}-\frac{\sin(x)}{a}$	14
default	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	30
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}-\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}-\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

input
```
int(sin(x)^2/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
(x-sin(x))/a
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx=\frac{x-\sin\left(x\right)}{a}$$

input
```
integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
(x - sin(x))/a
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(7) = 14.

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx=\frac{x\tan^2\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}+\frac{x}{a\tan^2\left(\frac{x}{2}\right)+a}-\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}$$

input
```
integrate(sin(x)**2/(a+a*cos(x)),x)
```

output
```
x*tan(x/2)**2/(a*tan(x/2)**2 + a) + x/(a*tan(x/2)**2 + a) - 2*tan(x/2)/(a*
tan(x/2)**2 + a)
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\,dx = \frac{2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a} - \frac{2\,\sin\left(x\right)}{\left(a + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos\left(x\right) + 1)}$$

input
```
integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
2*arctan(sin(x)/(cos(x) + 1))/a - 2*sin(x)/((a + a*sin(x)^2/(cos(x) + 1)^2
)*(cos(x) + 1))
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\,dx = \frac{x}{a} - \frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)a}$$

input
```
integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="giac")
```

output
```
x/a - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*a)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{\sin^2(x)}{a + a\cos(x)}\,dx = \frac{x - \sin(x)}{a}$$

input
```
int(sin(x)^2/(a + a*cos(x)),x)
```

output
```
(x - sin(x))/a
```

3.4 $\int \frac{\sin(x)}{a+a\cos(x)}\,dx$

3.4.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx = -\frac{\log(1+\cos(x))}{a}$$

output
```
-ln(1+cos(x))/a
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx = -\frac{2\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Sin[x]/(a + a*Cos[x]),x]
```

output
```
(-2*Log[Cos[x/2]])/a
```

3.4.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a\cos(x)+a}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a-a\sin\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 3146$$
$$-\frac{\int \frac{1}{\cos(x)a+a}d(a\cos(x))}{a}$$
$$\Big\downarrow \ 16$$
$$-\frac{\log(a\cos(x)+a)}{a}$$

input
```
Int[Sin[x]/(a + a*Cos[x]),x]
```

output
```
-(Log[a + a*Cos[x]]/a)
```

3.4.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.4.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-\frac{\ln(a+\cos(x)a)}{a}$	13
default	$-\frac{\ln(a+\cos(x)a)}{a}$	13
norman	$\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{a}$	14
parallelrisch	$\frac{\ln\left(\frac{2}{\cos(x)+1}\right)}{a}$	14
risch	$\frac{ix}{a}-\frac{2\ln\left(e^{ix}+1\right)}{a}$	22

input
```
int(sin(x)/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+cos(x)*a)/a
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a+a\cos(x)}\,dx=-\frac{\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-log(1/2*cos(x) + 1/2)/a
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(\cos\left(x\right) + 1\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x)
```

output
```
-log(cos(x) + 1)/a
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(a\cos\left(x\right) + a\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
-log(a*cos(x) + a)/a
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + a\cos(x)}\, dx = -\frac{\log\left(\cos\left(x\right) + 1\right)}{a}$$

input
```
integrate(sin(x)/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-log(cos(x) + 1)/a
```

3.4.9 Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a + a\cos(x)}\,dx = -\frac{\ln\left(\cos\left(x\right) + 1\right)}{a}$$

input
```
int(sin(x)/(a + a*cos(x)),x)
```

output
```
-log(cos(x) + 1)/a
```

3.5 $\int \frac{1}{a+a\cos(x)}\,dx$

3.5.1 Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\sin(x)}{a+a\cos(x)}$$

output
```
sin(x)/(a+a*cos(x))
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+a\cos(x)}\,dx=\frac{\tan\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[(a + a*Cos[x])^(-1),x]
```

output
```
Tan[x/2]/a
```

3.5.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(x)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a\sin\left(x+\frac{\pi}{2}\right)+a}dx$$
$$\Big\downarrow \text{3127}$$
$$\frac{\sin(x)}{a\cos(x)+a}$$

input
```
Int[(a + a*Cos[x])^(-1),x]
```

output
```
Sin[x]/(a + a*Cos[x])
```

3.5.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
norman	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
parallelrisch	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
risch	$\frac{2i}{\left(e^{ix}+1\right)a}$	16

input
```
int(1/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)/a
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\sin\left(x\right)}{a\cos\left(x\right)+a}$$

input
```
integrate(1/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
sin(x)/(a*cos(x) + a)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{a}$$

input
```
integrate(1/(a+a*cos(x)),x)
```

output
```
tan(x/2)/a
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\sin(x)}{a(\cos(x)+1)}$$

input
```
integrate(1/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
sin(x)/(a*(cos(x) + 1))
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\tan\left(\frac{1}{2}x\right)}{a}$$

input
```
integrate(1/(a+a*cos(x)),x, algorithm="giac")
```

output
```
tan(1/2*x)/a
```

3.5.9 Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{a}$$

input
```
int(1/(a + a*cos(x)),x)
```

output
```
tan(x/2)/a
```

3.6 $\int \frac{\csc(x)}{a+a\cos(x)}\,dx$

3.6.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = -\frac{\operatorname{arctanh}(\cos(x))}{2a}+\frac{1}{2(a+a\cos(x))}$$

output
```
-1/2*arctanh(cos(x))/a+1/2/(a+a*cos(x))
```

3.6.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = \frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cos(x))}$$

input
```
Integrate[Csc[x]/(a + a*Cos[x]),x]
```

output
```
(1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(2*a*(1 + Cos[x]))
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a\cos(x)+a}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \; 3146$$
$$-a\int \frac{1}{(a-a\cos(x))(\cos(x)a+a)^2}d(a\cos(x))$$
$$\Big\downarrow \; 54$$
$$-a\int \left(\frac{1}{2\left(a^2-a^2\cos^2(x)\right)a}+\frac{1}{2(\cos(x)a+a)^2a}\right)d(a\cos(x))$$
$$\Big\downarrow \; 2009$$
$$-a\left(\frac{\text{arctanh}(\cos(x))}{2a^2}-\frac{1}{2a(a\cos(x)+a)}\right)$$

input
```
Int[Csc[x]/(a + a*Cos[x]),x]
```

output
```
-(a*(ArcTanh[Cos[x]]/(2*a^2) - 1/(2*a*(a + a*Cos[x]))))
```

3.6.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.6.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{\tan^2\left(\frac{x}{2}\right)+2\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a}$	20
norman	$\frac{\tan^2\left(\frac{x}{2}\right)}{4a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	23
default	$\frac{\frac{1}{2\cos(x)+2}-\frac{\ln(\cos(x)+1)}{4}+\frac{\ln(\cos(x)-1)}{4}}{a}$	28
risch	$\frac{e^{ix}}{\left(e^{ix}+1\right)^2 a}-\frac{\ln\left(e^{ix}+1\right)}{2a}+\frac{\ln\left(e^{ix}-1\right)}{2a}$	46

input
```
int(csc(x)/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(tan(1/2*x)^2+2*ln(tan(1/2*x)))/a
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx = -\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{4\,(a\cos(x)+a)}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) +
1/2) - 2)/(a*cos(x) + a)
```

3.6.6 Sympy [F]

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = \frac{\int \frac{\csc(x)}{\cos(x)+1}\,dx}{a}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)/(cos(x) + 1), x)/a
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = -\frac{\log(\cos(x)+1)}{4\,a} + \frac{\log(\cos(x)-1)}{4\,a} + \frac{1}{2\,(a\cos(x)+a)}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
-1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/a + 1/2/(a*cos(x) + a)
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{\csc(x)}{a + a\cos(x)}\,dx = -\frac{\log(\cos(x)+1)}{4\,a} + \frac{\log(-\cos(x)+1)}{4\,a} + \frac{1}{2\,a(\cos(x)+1)}$$

input
```
integrate(csc(x)/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a + 1/2/(a*(cos(x) + 1))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\csc(x)}{a+a\cos(x)}\,dx=\frac{1}{2\,a\,\left(\cos\left(x\right)+1\right)}-\frac{\mathrm{atanh}(\cos\left(x\right))}{2\,a}$$

input
```
int(1/(sin(x)*(a + a*cos(x))),x)
```

output
```
1/(2*a*(cos(x) + 1)) - atanh(cos(x))/(2*a)
```

3.7 $\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx$

3.7.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx=-\frac{2\cot(x)}{3a}+\frac{\csc(x)}{3(a+a\cos(x))}$$

output
```
-2/3*cot(x)/a+1/3*csc(x)/(a+a*cos(x))
```

3.7.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx=-\frac{(2\cos(x)+\cos(2x))\csc\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)}{12a}$$

input
```
Integrate[Csc[x]^2/(a + a*Cos[x]),x]
```

output
```
-1/12*((2*Cos[x] + Cos[2*x])*Csc[x/2]*Sec[x/2]^3)/a
```

3.7.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^2\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{2\int \csc^2(x)dx}{3a}+\frac{\csc(x)}{3(a\cos(x)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \csc(x)^2dx}{3a}+\frac{\csc(x)}{3(a\cos(x)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\csc(x)}{3(a\cos(x)+a)}-\frac{2\int 1d\cot(x)}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\csc(x)}{3(a\cos(x)+a)}-\frac{2\cot(x)}{3a}$$

input
```
Int[Csc[x]^2/(a + a*Cos[x]),x]
```

output
```
(-2*Cot[x])/(3*a) + Csc[x]/(3*(a + a*Cos[x]))
```

3.7.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{\tan^3\left(\frac{x}{2}\right)+6\tan\left(\frac{x}{2}\right)-3\cot\left(\frac{x}{2}\right)}{12a}$	25
default	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+2\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}}{4a}$	29
risch	$-\frac{4i\left(1+2\,\mathrm{e}^{ix}\right)}{3\left(\mathrm{e}^{ix}+1\right)^3 a\left(\mathrm{e}^{ix}-1\right)}$	34
norman	$\frac{-\frac{1}{4a}+\frac{\tan^2\left(\frac{x}{2}\right)}{2a}+\frac{\tan^4\left(\frac{x}{2}\right)}{12a}}{\tan\left(\frac{x}{2}\right)}$	36

input
```
int(csc(x)^2/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(tan(1/2*x)^3+6*tan(1/2*x)-3*cot(1/2*x))/a
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = -\frac{2\,\cos(x)^2 + 2\,\cos(x) - 1}{3\,(a\cos(x)+a)\sin(x)}$$

input
```
integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-1/3*(2*cos(x)^2 + 2*cos(x) - 1)/((a*cos(x) + a)*sin(x))
```

3.7.6 Sympy [F]

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{\csc^2(x)}{\cos(x)+1}\,dx}{a}$$

input
```
integrate(csc(x)**2/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)**2/(cos(x) + 1), x)/a
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = \frac{\frac{6\,\sin(x)}{\cos(x)+1} + \frac{\sin(x)^3}{(\cos(x)+1)^3}}{12\,a} - \frac{\cos(x)+1}{4\,a\sin(x)}$$

input
```
integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/12*(6*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a - 1/4*(cos(x) + 1
)/(a*sin(x))
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\, dx = \frac{a^2 \tan\left(\frac{1}{2}\,x\right)^3 + 6\,a^2 \tan\left(\frac{1}{2}\,x\right)}{12\,a^3} - \frac{1}{4\,a \tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="giac")
```

output
```
1/12*(a^2*tan(1/2*x)^3 + 6*a^2*tan(1/2*x))/a^3 - 1/4/(a*tan(1/2*x))
```

3.7.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\csc^2(x)}{a + a\cos(x)}\, dx = \frac{-8\cos\left(\frac{x}{2}\right)^4 + 4\cos\left(\frac{x}{2}\right)^2 + 1}{12\,a\cos\left(\frac{x}{2}\right)^3 \sin\left(\frac{x}{2}\right)}$$

input
```
int(1/(sin(x)^2*(a + a*cos(x))),x)
```

output
```
(4*cos(x/2)^2 - 8*cos(x/2)^4 + 1)/(12*a*cos(x/2)^3*sin(x/2))
```

3.8 $\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx$

3.8.1 Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx=-\frac{3\text{arctanh}(\cos(x))}{8a}-\frac{1}{8(a-a\cos(x))}+\frac{a}{8(a+a\cos(x))^2}+\frac{1}{4(a+a\cos(x))}$$

output
```
-3/8*arctanh(cos(x))/a-1/8/(a-a*cos(x))+1/8*a/(a+a*cos(x))^2+1/4/(a+a*cos(
x))
```

3.8.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx=\frac{4-2\cot^2\left(\frac{x}{2}\right)-12\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\sec^2\left(\frac{x}{2}\right)}{16a(1+\cos(x))}$$

input
```
Integrate[Csc[x]^3/(a + a*Cos[x]),x]
```

output
```
(4 - 2*Cot[x/2]^2 - 12*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]) + Sec[x/
2]^2)/(16*a*(1 + Cos[x]))
```

3.8.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{a\cos(x)+a}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^3\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \ 3146$$
$$-a^3\int \frac{1}{(a-a\cos(x))^2(\cos(x)a+a)^3}d(a\cos(x))$$
$$\Big\downarrow \ 54$$
$$-a^3\int \left(\frac{1}{8a^3(a-a\cos(x))^2}+\frac{1}{4a^3(\cos(x)a+a)^2}+\frac{1}{4a^2(\cos(x)a+a)^3}+\frac{3}{8a^3\left(a^2-a^2\cos^2(x)\right)}\right)d(a\cos(x))$$
$$\Big\downarrow \ 2009$$
$$-a^3\left(\frac{3\text{arctanh}(\cos(x))}{8a^4}+\frac{1}{8a^3(a-a\cos(x))}-\frac{1}{4a^3(a\cos(x)+a)}-\frac{1}{8a^2(a\cos(x)+a)^2}\right)$$

input
```
Int[Csc[x]^3/(a + a*Cos[x]),x]
```

output
```
-(a^3*((3*ArcTanh[Cos[x]])/(8*a^4) + 1/(8*a^3*(a - a*Cos[x])) - 1/(8*a^2*(
a + a*Cos[x])^2) - 1/(4*a^3*(a + a*Cos[x]))))
```

3.8.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.8.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$\frac{\tan^4\left(\frac{x}{2}\right)+6\left(\tan^2\left(\frac{x}{2}\right)\right)-2\left(\cot^2\left(\frac{x}{2}\right)\right)+12\ln\left(\tan\left(\frac{x}{2}\right)\right)}{32a}$	36
default	$\frac{\frac{1}{8\cos(x)-8}+\frac{3\ln(\cos(x)-1)}{16}+\frac{1}{8(\cos(x)+1)^2}+\frac{1}{4\cos(x)+4}-\frac{3\ln(\cos(x)+1)}{16}}{a}$	44
norman	$\frac{-\frac{1}{16a}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{16a}+\frac{\tan^6\left(\frac{x}{2}\right)}{32a}}{\tan\left(\frac{x}{2}\right)^2}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8a}$	47
risch	$\frac{3\,e^{5ix}+6\,e^{4ix}-2\,e^{3ix}+6\,e^{2ix}+3\,e^{ix}}{4(e^{ix}+1)^4a(e^{ix}-1)^2}-\frac{3\ln(e^{ix}+1)}{8a}+\frac{3\ln(e^{ix}-1)}{8a}$	87

input
```
int(csc(x)^3/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/32*(tan(1/2*x)^4+6*tan(1/2*x)^2-2*cot(1/2*x)^2+12*ln(tan(1/2*x)))/a
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx$$
$$= \frac{6\cos(x)^2 - 3\left(\cos(x)^3+\cos(x)^2-\cos(x)-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^3+\cos(x)^2-\cos(x)-1\right.}{16\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)-a\right)}$$

input
```
integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
1/16*(6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(1/2*cos(x) + 1
/2) + 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(-1/2*cos(x) + 1/2) + 6*cos(
x) - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a)
```

3.8.6 Sympy [F]

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{\csc^3(x)}{\cos(x)+1}\,dx}{a}$$

input
```
integrate(csc(x)**3/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)**3/(cos(x) + 1), x)/a
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = \frac{3\cos(x)^2+3\cos(x)-2}{8\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)-a\right)} - \frac{3\log(\cos(x)+1)}{16\,a} + \frac{3\log(\cos(x)-1)}{16\,a}$$

input
```
integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/8*(3*cos(x)^2 + 3*cos(x) - 2)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a) -
 3/16*log(cos(x) + 1)/a + 3/16*log(cos(x) - 1)/a
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\,dx = -\frac{3\,\log\left(\cos\left(x\right) + 1\right)}{16\,a} + \frac{3\,\log\left(-\cos\left(x\right) + 1\right)}{16\,a} + \frac{3\,\cos\left(x\right)^{2} + 3\,\cos\left(x\right) - 2}{8\,a{\left(\cos\left(x\right) + 1\right)}^{2}{\left(\cos\left(x\right) - 1\right)}}$$

input
```
integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-3/16*log(cos(x) + 1)/a + 3/16*log(-cos(x) + 1)/a + 1/8*(3*cos(x)^2 + 3*co
s(x) - 2)/(a*(cos(x) + 1)^2*(cos(x) - 1))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{\csc^3(x)}{a + a\cos(x)}\,dx = -\frac{\frac{3\cos(x)^2}{8} + \frac{3\,\cos(x)}{8} - \frac{1}{4}}{-a\cos\left(x\right)^3 - a\cos\left(x\right)^2 + a\,\cos\left(x\right) + a} - \frac{3\,\mathrm{atanh}(\cos\left(x\right))}{8\,a}$$

input
```
int(1/(sin(x)^3*(a + a*cos(x))),x)
```

output
```
- ((3*cos(x))/8 + (3*cos(x)^2)/8 - 1/4)/(a + a*cos(x) - a*cos(x)^2 - a*cos
(x)^3) - (3*atanh(cos(x)))/(8*a)
```

3.9 $\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx$

3.9.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx=-\frac{4\cot(x)}{5a}-\frac{4\cot^3(x)}{15a}+\frac{\csc^3(x)}{5(a+a\cos(x))}$$

output
```
-4/5*cot(x)/a-4/15*cot(x)^3/a+1/5*csc(x)^3/(a+a*cos(x))
```

3.9.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx=\frac{(-6\cos(x)-2\cos(2x)+2\cos(3x)+\cos(4x))\csc^3(x)}{15a(1+\cos(x))}$$

input
```
Integrate[Csc[x]^4/(a + a*Cos[x]),x]
```

output
```
((-6*Cos[x] - 2*Cos[2*x] + 2*Cos[3*x] + Cos[4*x])*Csc[x]^3)/(15*a*(1 + Cos
[x]))
```

3.9.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^4\left(a-a\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{4\int \csc^4(x)dx}{5a}+\frac{\csc^3(x)}{5(a\cos(x)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \csc(x)^4dx}{5a}+\frac{\csc^3(x)}{5(a\cos(x)+a)}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\csc^3(x)}{5(a\cos(x)+a)}-\frac{4\int \left(\cot^2(x)+1\right)d\cot(x)}{5a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\csc^3(x)}{5(a\cos(x)+a)}-\frac{4\left(\frac{\cot^3(x)}{3}+\cot(x)\right)}{5a}$$

input
```
Int[Csc[x]^4/(a + a*Cos[x]),x]
```

output
```
(-4*(Cot[x] + Cot[x]^3/3))/(5*a) + Csc[x]^3/(5*(a + a*Cos[x]))
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+6\tan\left(\frac{x}{2}\right)-\frac{4}{\tan\left(\frac{x}{2}\right)}-\frac{1}{3\tan\left(\frac{x}{2}\right)^3}}{16a}$	45
risch	$\frac{16i\left(6\,\mathrm{e}^{3ix}+2\,\mathrm{e}^{2ix}-2\,\mathrm{e}^{ix}-1\right)}{15\left(\mathrm{e}^{ix}-1\right)^3 a\left(\mathrm{e}^{ix}+1\right)^5}$	48
parallelrisch	$-\frac{4\csc(x)(-\cos(4x)+2\cos(2x)-2\cos(3x)+6\cos(x))}{15a(\cos(x)-\cos(3x)-2\cos(2x)+2)}$	49
norman	$\frac{-\frac{1}{48a}-\frac{\tan^2\left(\frac{x}{2}\right)}{4a}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^6\left(\frac{x}{2}\right)}{12a}+\frac{\tan^8\left(\frac{x}{2}\right)}{80a}}{\tan\left(\frac{x}{2}\right)^3}$	58

input
```
int(csc(x)^4/(a+cos(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*(1/5*tan(1/2*x)^5+4/3*tan(1/2*x)^3+6*tan(1/2*x)-4/tan(1/2*x)-1/3/ta
n(1/2*x)^3)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = -\frac{8\cos(x)^4+8\cos(x)^3-12\cos(x)^2-12\cos(x)+3}{15\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)-a\right)\sin(x)}$$

input
```
integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
-1/15*(8*cos(x)^4 + 8*cos(x)^3 - 12*cos(x)^2 - 12*cos(x) + 3)/((a*cos(x)^3
 + a*cos(x)^2 - a*cos(x) - a)*sin(x))
```

3.9.6 Sympy [F]

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{\csc^4(x)}{\cos(x)+1}\,dx}{a}$$

input
```
integrate(csc(x)**4/(a+a*cos(x)),x)
```

output
```
Integral(csc(x)**4/(cos(x) + 1), x)/a
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx = \frac{\frac{90\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}}{240\,a} - \frac{\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)^3}{48\,a\sin(x)^3}$$

input
```
integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
1/240*(90*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5/(c
os(x) + 1)^5)/a - 1/48*(12*sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)^3/(a*
sin(x)^3)
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{\csc^4(x)}{a + a\cos(x)}\,dx = -\frac{12\tan\left(\frac{1}{2}x\right)^2 + 1}{48\,a\tan\left(\frac{1}{2}x\right)^3} + \frac{3\,a^4\tan\left(\frac{1}{2}x\right)^5 + 20\,a^4\tan\left(\frac{1}{2}x\right)^3 + 90\,a^4\tan\left(\frac{1}{2}x\right)}{240\,a^5}$$

input
```
integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

output
```
-1/48*(12*tan(1/2*x)^2 + 1)/(a*tan(1/2*x)^3) + 1/240*(3*a^4*tan(1/2*x)^5 +
 20*a^4*tan(1/2*x)^3 + 90*a^4*tan(1/2*x))/a^5
```

3.9.9 Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{\csc^4(x)}{a + a\cos(x)}\,dx = \frac{3\tan\left(\frac{x}{2}\right)^8 + 20\tan\left(\frac{x}{2}\right)^6 + 90\tan\left(\frac{x}{2}\right)^4 - 60\tan\left(\frac{x}{2}\right)^2 - 5}{240\,a\tan\left(\frac{x}{2}\right)^3}$$

input
```
int(1/(sin(x)^4*(a + a*cos(x))),x)
```

output
```
(90*tan(x/2)^4 - 60*tan(x/2)^2 + 20*tan(x/2)^6 + 3*tan(x/2)^8 - 5)/(240*a*
tan(x/2)^3)
```

3.10 $\int \frac{\sin(2x)}{1+\cos(2x)}\,dx$

3.10.1 Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\log(\cos(x))$$

output
```
-ln(cos(x))
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\log(\cos(x))$$

input
```
Integrate[Sin[2*x]/(1 + Cos[2*x]),x]
```

output
```
-Log[Cos[x]]
```

3.10.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\cos(2x)+1}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos\left(2x-\frac{\pi}{2}\right)}{1-\sin\left(2x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 3146$$
$$-\frac{1}{2}\int \frac{1}{\cos(2x)+1}d\cos(2x)$$
$$\Big\downarrow \ 16$$
$$-\frac{1}{2}\log(\cos(2x)+1)$$

input
```
Int[Sin[2*x]/(1 + Cos[2*x]),x]
```

output
```
-1/2*Log[1 + Cos[2*x]]
```

3.10.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.10.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

method	result	size
parallelrisch	$\ln\left(\sqrt{\sec^{2}\left(x\right)}\right)$	8
derivativedivides	$-\frac{\ln(1+\cos(2x))}{2}$	10
default	$-\frac{\ln(1+\cos(2x))}{2}$	10
norman	$\frac{\ln(1+\tan^{2}(x))}{2}$	10
risch	$ix-\ln\left(\mathrm{e}^{2ix}+1\right)$	16

input
```
int(sin(2*x)/(1+cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
ln((sec(x)^2)^(1/2))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(2*x) + 1/2)
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{\log(\cos(2x)+1)}{2}$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x)
```

output
```
-log(cos(2*x) + 1)/2
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\log(\cos(2x)+1)$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="maxima")
```

output
```
-1/2*log(cos(2*x) + 1)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{1}{2}\log(\cos(2x)+1)$$

input
```
integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="giac")
```

output
```
-1/2*log(cos(2*x) + 1)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \frac{\sin(2x)}{1+\cos(2x)}\,dx = -\frac{\ln\left(\cos\left(x\right)^2\right)}{2}$$

input
```
int(sin(2*x)/(cos(2*x) + 1),x)
```

output
```
-log(cos(x)^2)/2
```

3.11 $\int \frac{\sin(2x)}{1-\cos(2x)}\,dx$

3.11.1 Optimal result

Integrand size = 15, antiderivative size = 3

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \log(\sin(x))$$

output
```
ln(sin(x))
```

3.11.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 2.33

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \log(\cos(x)) + \log(\tan(x))$$

input
```
Integrate[Sin[2*x]/(1 - Cos[2*x]),x]
```

output
```
Log[Cos[x]] + Log[Tan[x]]
```

3.11.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 13 vs. $2(3) = 6$.

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 4.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{1-\cos(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(2x-\frac{\pi}{2}\right)}{\sin\left(2x-\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3146}$$

$$\frac{1}{2}\int \frac{1}{1-\cos(2x)}d(-\cos(2x))$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\log(1-\cos(2x))$$

input
```
Int[Sin[2*x]/(1 - Cos[2*x]),x]
```

output
```
Log[1 - Cos[2*x]]/2
```

3.11.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 4.00

method	result	size
derivativedivides	$\frac{\ln(1-\cos(2x))}{2}$	12
default	$\frac{\ln(1-\cos(2x))}{2}$	12
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right)+\ln(\tan(x))$	12
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
risch	$-ix+\ln(e^{2ix}-1)$	14

input
```
int(sin(2*x)/(1-cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1-cos(2*x))
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="fricas")
```

output
```
1/2*log(-1/2*cos(2*x) + 1/2)
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{\log(\cos(2x)-1)}{2}$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x)
```

output
```
log(cos(2*x) - 1)/2
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9 vs. $2(3) = 6$.

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\log(\cos(2x)-1)$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="maxima")
```

output
```
1/2*log(cos(2*x) - 1)
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 3.67

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{1}{2}\log(-\cos(2x)+1)$$

input
```
integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="giac")
```

output
```
1/2*log(-cos(2*x) + 1)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\sin(2x)}{1-\cos(2x)}\,dx = \frac{\ln\left(-\sin\left(x\right)^2\right)}{2}$$

input
```
int(-sin(2*x)/(cos(2*x) - 1),x)
```

output
```
log(-sin(x)^2)/2
```

3.12 $\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx$

3.12.1 Optimal result

Integrand size = 9, antiderivative size = 6

$$\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx = \frac{1}{1+\cos(x)}$$

output
```
1/(1+cos(x))
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2} \, dx = \frac{1}{2} \sec^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 + Cos[x])^2,x]
```

output
```
Sec[x/2]^2/2
```

3.12.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(\cos(x)+1)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3146$$

$$-\int \frac{1}{(\cos(x)+1)^2}d\cos(x)$$

$$\Big\downarrow \; 17$$

$$\frac{1}{\cos(x)+1}$$

input
```
Int[Sin[x]/(1 + Cos[x])^2,x]
```

output
```
(1 + Cos[x])^(-1)
```

3.12.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.12.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{1}{\cos(x)+1}$	7
default	$\frac{1}{\cos(x)+1}$	7
norman	$\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}$	9
parallelrisch	$\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}$	9
risch	$\frac{2\,e^{ix}}{\left(e^{ix}+1\right)^2}$	17

input
```
int(sin(x)/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(cos(x)+1)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+1}$$

input
```
integrate(sin(x)/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
1/(cos(x) + 1)
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+1}$$

input
```
integrate(sin(x)/(1+cos(x))**2,x)
```

output
```
1/(cos(x) + 1)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+1}$$

input
```
integrate(sin(x)/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
1/(cos(x) + 1)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx = \frac{1}{\cos{(x)}+1}$$

input
```
integrate(sin(x)/(1+cos(x))^2,x, algorithm="giac")
```

output
```
1/(cos(x) + 1)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1+\cos(x))^2}\, dx = \frac{1}{\cos(x)+1}$$

input
```
int(sin(x)/(cos(x) + 1)^2,x)
```

output
```
1/(cos(x) + 1)
```

3.13 $\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx$

3.13.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx = -\frac{1}{1-\cos(x)}$$

output
```
-1/(1-cos(x))
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx = -\frac{1}{2}\csc^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 - Cos[x])^2,x]
```

output
```
-1/2*Csc[x/2]^2
```

3.13.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx$$
$$\Big\downarrow \; 3146$$
$$\int \frac{1}{(1-\cos(x))^2}d(-\cos(x))$$
$$\Big\downarrow \; 17$$
$$-\frac{1}{1-\cos(x)}$$

input
```
Int[Sin[x]/(1 - Cos[x])^2,x]
```

output
```
-(1 - Cos[x])^(-1)
```

3.13.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.13.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$-\frac{1}{2\tan\left(\frac{x}{2}\right)^{2}}$	9
derivativedivides	$-\frac{1}{1-\cos(x)}$	11
default	$-\frac{1}{1-\cos(x)}$	11
risch	$\frac{2\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}-1\right)^{2}}$	17
norman	$\frac{-\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2}-\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^{3}}$	33

input
```
int(sin(x)/(1-cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/tan(1/2*x)^2
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

input
```
integrate(sin(x)/(1-cos(x))^2,x, algorithm="fricas")
```

output
```
1/(cos(x) - 1)
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

input
```
integrate(sin(x)/(1-cos(x))**2,x)
```

output
```
1/(cos(x) - 1)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

input
```
integrate(sin(x)/(1-cos(x))^2,x, algorithm="maxima")
```

output
```
1/(cos(x) - 1)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

input
```
integrate(sin(x)/(1-cos(x))^2,x, algorithm="giac")
```

output
```
1/(cos(x) - 1)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sin(x)}{(1-\cos(x))^2}\,dx = \frac{1}{\cos(x)-1}$$

input
```
int(sin(x)/(cos(x) - 1)^2,x)
```

output
```
1/(cos(x) - 1)
```

3.14 $\int \frac{\sin^2(x)}{(1+\cos(x))^2} dx$

3.14.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} dx = -x + \frac{2\sin(x)}{1+\cos(x)}$$

output
```
-x+2*sin(x)/(1+cos(x))
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} dx = -2\arctan\left(\tan\left(\frac{x}{2}\right)\right) + 2\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^2/(1 + Cos[x])^2,x]
```

output
```
-2*ArcTan[Tan[x/2]] + 2*Tan[x/2]
```

3.14.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(\cos(x)+1)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3159$$

$$\frac{2\sin(x)}{\cos(x)+1}-\int 1dx$$

$$\Big\downarrow \; 24$$

$$\frac{2\sin(x)}{\cos(x)+1}-x$$

input
```
Int[Sin[x]^2/(1 + Cos[x])^2,x]
```

output
```
-x + (2*Sin[x])/(1 + Cos[x])
```

3.14.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.14.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$-x+2\tan\left(\frac{x}{2}\right)$	11
default	$2\tan\left(\frac{x}{2}\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	15
risch	$-x+\frac{4i}{e^{ix}+1}$	17
norman	$\frac{-x+4\left(\tan^{3}\left(\frac{x}{2}\right)\right)+2\left(\tan^{5}\left(\frac{x}{2}\right)\right)-2\left(\tan^{2}\left(\frac{x}{2}\right)\right)x-\left(\tan^{4}\left(\frac{x}{2}\right)\right)x+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	56

input
```
int(sin(x)^2/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+2*tan(1/2*x)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx=-\frac{x\cos(x)+x-2\sin(x)}{\cos(x)+1}$$

input
```
integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = -x + 2\tan\left(\frac{x}{2}\right)$$

input
```
integrate(sin(x)**2/(1+cos(x))**2,x)
```

output
```
-x + 2*tan(x/2)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = \frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1} - 2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

input
```
integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx = -x + 2\,\tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="giac")
```

output
```
-x + 2*tan(1/2*x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2(x)}{(1+\cos(x))^2} \, dx = 2\tan\left(\frac{x}{2}\right) - x$$

input
```
int(sin(x)^2/(cos(x) + 1)^2,x)
```

output
```
2*tan(x/2) - x
```

3.15 $\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx$

3.15.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx = -x - \frac{2\sin(x)}{1-\cos(x)}$$

output
```
-x-2*sin(x)/(1-cos(x))
```

3.15.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} dx = -2\cot\left(\frac{x}{2}\right) \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^2/(1 - Cos[x])^2,x]
```

output
```
-2*Cot[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x/2]^2]
```

3.15.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \; 3159$$

$$-\int 1dx-\frac{2\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \; 24$$

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

input
```
Int[Sin[x]^2/(1 - Cos[x])^2,x]
```

output
```
-x - (2*Sin[x])/(1 - Cos[x])
```

3.15.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

3.15.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	17
risch	$-x-\frac{4i}{e^{ix}-1}$	17
parallelrisch	$\frac{-\tan\left(\frac{x}{2}\right)x-2}{\tan\left(\frac{x}{2}\right)}$	17
norman	$\frac{-2\left(\tan^2\left(\frac{x}{2}\right)\right)-4\left(\tan^4\left(\frac{x}{2}\right)\right)-2\left(\tan^6\left(\frac{x}{2}\right)\right)-x\left(\tan^7\left(\frac{x}{2}\right)\right)-\left(\tan^3\left(\frac{x}{2}\right)\right)x-2\left(\tan^5\left(\frac{x}{2}\right)\right)x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)^3}$	70

input
```
int(sin(x)^2/(1-cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/tan(1/2*x)-2*arctan(tan(1/2*x))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx=-\frac{x\sin\left(x\right)+2\,\cos\left(x\right)+2}{\sin\left(x\right)}$$

input
```
integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="fricas")
```

output
```
-(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\, dx = -x - \frac{2}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate(sin(x)**2/(1-cos(x))**2,x)
```

output
```
-x - 2/tan(x/2)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\, dx = -\frac{2\,(\cos(x)+1)}{\sin(x)} - 2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="maxima")
```

output
```
-2*(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\, dx = -x - \frac{2}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="giac")
```

output
```
-x - 2/tan(1/2*x)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx = -x - 2\cot\left(\frac{x}{2}\right)$$

input
```
int(sin(x)^2/(cos(x) - 1)^2,x)
```

output
```
- x - 2*cot(x/2)
```

3.16 $\int \frac{\sin^3(x)}{(1+\cos(x))^2} dx$

3.16.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2} dx = \cos(x) - 2\log(1+\cos(x))$$

output
```
cos(x)-2*ln(1+cos(x))
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2} dx = -1 + \cos(x) - 4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^3/(1 + Cos[x])^2,x]
```

output
```
-1 + Cos[x] - 4*Log[Cos[x/2]]
```

3.16.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(\cos(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1-\cos(x)}{\cos(x)+1}d\cos(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{\cos(x)+1}-1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)-2\log(\cos(x)+1)$$

input
```
Int[Sin[x]^3/(1 + Cos[x])^2,x]
```

output
```
Cos[x] - 2*Log[1 + Cos[x]]
```

3.16.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.16.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\cos(x) - 2\ln(\cos(x)+1)$	11
default	$\cos(x) - 2\ln(\cos(x)+1)$	11
parallelrisch	$2\ln\left(\frac{1}{\cos(x)+1}\right) + \ln(4) + \cos(x) + 1$	16
risch	$2ix + \frac{e^{ix}}{2} + \frac{e^{-ix}}{2} - 4\ln(e^{ix}+1)$	30
norman	$\frac{2(\tan^4(\frac{x}{2}))+4(\tan^2(\frac{x}{2}))+2}{(1+\tan^2(\frac{x}{2}))^3} + 2\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	42

input
```
int(sin(x)^3/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

output
```
cos(x)-2*ln(cos(x)+1)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\,\log\left(\frac{1}{2}\,\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="fricas")
```

output
```
cos(x) - 2*log(1/2*cos(x) + 1/2)
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = -\frac{2\log(\cos(x)+1)\cos(x)}{\cos(x)+1} - \frac{2\log(\cos(x)+1)}{\cos(x)+1} + \frac{\sin^2(x)}{\cos(x)+1} + \frac{2\cos^2(x)}{\cos(x)+1} - \frac{2}{\cos(x)+1}$$

input
```
integrate(sin(x)**3/(1+cos(x))**2,x)
```

output
```
-2*log(cos(x) + 1)*cos(x)/(cos(x) + 1) - 2*log(cos(x) + 1)/(cos(x) + 1) +
sin(x)**2/(cos(x) + 1) + 2*cos(x)**2/(cos(x) + 1) - 2/(cos(x) + 1)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\log(\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="maxima")
```

output
```
cos(x) - 2*log(cos(x) + 1)
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\log(\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="giac")
```

output
```
cos(x) - 2*log(cos(x) + 1)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^2}\,dx = \cos(x) - 2\ln(\cos(x) + 1)$$

input
```
int(sin(x)^3/(cos(x) + 1)^2,x)
```

output
```
cos(x) - 2*log(cos(x) + 1)
```

3.17 $\int \frac{\sin^3(x)}{(1-\cos(x))^2} dx$

3.17.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2} dx = \cos(x) + 2\log(1-\cos(x))$$

output
```
cos(x)+2*ln(1-cos(x))
```

3.17.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2} dx = -1 + \cos(x) + 4\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^3/(1 - Cos[x])^2,x]
```

output
```
-1 + Cos[x] + 4*Log[Sin[x/2]]
```

3.17.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2} \, dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx$$
$$\Big\downarrow \ 3146$$
$$\int \frac{\cos(x)+1}{1-\cos(x)}d(-\cos(x))$$
$$\Big\downarrow \ 49$$
$$\int \left(\frac{2}{1-\cos(x)}-1\right)d(-\cos(x))$$
$$\Big\downarrow \ 2009$$
$$\cos(x)+2\log(1-\cos(x))$$

input
```
Int[Sin[x]^3/(1 - Cos[x])^2,x]
```

output
```
Cos[x] + 2*Log[1 - Cos[x]]
```

3.17.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.17.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\cos(x)+2\ln(\cos(x)-1)$	11
default	$\cos(x)+2\ln(\cos(x)-1)$	11
parallelrisch	$4\ln(\csc(x)-\cot(x))-2\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{1}{4}\right)+\cos(x)+1$	26
risch	$-2ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+4\ln(e^{ix}-1)$	30
norman	$\frac{2(\tan^3(\frac{x}{2}))+2(\tan^7(\frac{x}{2}))+4(\tan^5(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^3\tan(\frac{x}{2})^3}+4\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	62

input
```
int(sin(x)^3/(1-cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
cos(x)+2*ln(cos(x)-1)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx=\cos(x)+2\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="fricas")
```

output
```
cos(x) + 2*log(-1/2*cos(x) + 1/2)
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \frac{2\log(\cos(x)-1)\cos(x)}{\cos(x)-1} - \frac{2\log(\cos(x)-1)}{\cos(x)-1} + \frac{\sin^2(x)}{\cos(x)-1} + \frac{2\cos^2(x)}{\cos(x)-1} - \frac{2}{\cos(x)-1}$$

input
```
integrate(sin(x)**3/(1-cos(x))**2,x)
```

output
```
2*log(cos(x) - 1)*cos(x)/(cos(x) - 1) - 2*log(cos(x) - 1)/(cos(x) - 1) + s
in(x)**2/(cos(x) - 1) + 2*cos(x)**2/(cos(x) - 1) - 2/(cos(x) - 1)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos(x) + 2\log(\cos(x)-1)$$

input
```
integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="maxima")
```

output
```
cos(x) + 2*log(cos(x) - 1)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = \cos(x) + 2\log(-\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="giac")
```

output
```
cos(x) + 2*log(-cos(x) + 1)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(x)}{(1-\cos(x))^2}\,dx = 2\,\ln\left(\cos\left(x\right)-1\right)+\cos\left(x\right)$$

input
```
int(sin(x)^3/(cos(x) - 1)^2,x)
```

output
```
2*log(cos(x) - 1) + cos(x)
```

3.18 $\int \frac{\sin(x)}{(1+\cos(x))^3} dx$

3.18.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\sin(x)}{(1+\cos(x))^3} dx = \frac{1}{2(1+\cos(x))^2}$$

output
```
1/2/(1+cos(x))^2
```

3.18.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sin(x)}{(1+\cos(x))^3} dx = \frac{1}{8}\sec^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 + Cos[x])^3,x]
```

output
```
Sec[x/2]^4/8
```

3.18.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(\cos(x)+1)^3} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 3146$$
$$-\int \frac{1}{(\cos(x)+1)^3}d\cos(x)$$
$$\Big\downarrow \; 17$$
$$\frac{1}{2(\cos(x)+1)^2}$$

input
```
Int[Sin[x]/(1 + Cos[x])^3,x]
```

output
```
1/(2*(1 + Cos[x])^2)
```

3.18.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.18.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{1}{2(\cos(x)+1)^2}$	9
default	$\frac{1}{2(\cos(x)+1)^2}$	9
risch	$\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{ix}+1)^4}$	17
parallelrisch	$\frac{(\tan^2(\frac{x}{2}))(\tan^2(\frac{x}{2})+2)}{8}$	17
norman	$\frac{(\tan^2(\frac{x}{2}))}{4}+\frac{(\tan^4(\frac{x}{2}))}{8}$	18

input
```
int(sin(x)/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/(cos(x)+1)^2
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos(x)^2+2\cos(x)+1\right)}$$

input
```
integrate(sin(x)/(1+cos(x))^3,x, algorithm="fricas")
```

output
```
1/2/(cos(x)^2 + 2*cos(x) + 1)
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\cos^2(x) + 4\cos(x) + 2}$$

input
```
integrate(sin(x)/(1+cos(x))**3,x)
```

output
```
1/(2*cos(x)**2 + 4*cos(x) + 2)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos(x) + 1\right)^2}$$

input
```
integrate(sin(x)/(1+cos(x))^3,x, algorithm="maxima")
```

output
```
1/2/(cos(x) + 1)^2
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos(x) + 1\right)^2}$$

input
```
integrate(sin(x)/(1+cos(x))^3,x, algorithm="giac")
```

output
```
1/2/(cos(x) + 1)^2
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx = \frac{1}{2\left(\cos\left(x\right)+1\right)^2}$$

input
```
int(sin(x)/(cos(x) + 1)^3,x)
```

output
```
1/(2*(cos(x) + 1)^2)
```

3.19 $\int \frac{\sin(x)}{(1-\cos(x))^3} dx$

3.19.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{(1-\cos(x))^3} dx = -\frac{1}{2(1-\cos(x))^2}$$

output
```
-1/2/(1-cos(x))^2
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{(1-\cos(x))^3} dx = -\frac{1}{8}\csc^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]/(1 - Cos[x])^3,x]
```

output
```
-1/8*Csc[x/2]^4
```

3.19.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(1-\cos(x))^3} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \; 3146$$

$$\int \frac{1}{(1-\cos(x))^3}d(-\cos(x))$$

$$\Big\downarrow \; 17$$

$$-\frac{1}{2(1-\cos(x))^2}$$

input
```
Int[Sin[x]/(1 - Cos[x])^3,x]
```

output
```
-1/2*1/(1 - Cos[x])^2
```

3.19.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.19.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{2(1-\cos(x))^2}$	11
default	$-\frac{1}{2(1-\cos(x))^2}$	11
risch	$-\frac{2\,e^{2ix}}{(e^{ix}-1)^4}$	17
parallelrisch	$-\frac{\left(\cot^2\left(\frac{x}{2}\right)\right)\left(\cot^2\left(\frac{x}{2}\right)+2\right)}{8}$	17
norman	$\frac{-\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{8}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)^5}$	41

input
```
int(sin(x)/(1-cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(1-cos(x))^2
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\left(\cos\left(x\right)^2-2\,\cos\left(x\right)+1\right)}$$

input
```
integrate(sin(x)/(1-cos(x))^3,x, algorithm="fricas")
```

output
```
-1/2/(cos(x)^2 - 2*cos(x) + 1)
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2\cos^2(x) - 4\cos(x) + 2}$$

input
```
integrate(sin(x)/(1-cos(x))**3,x)
```

output
```
-1/(2*cos(x)**2 - 4*cos(x) + 2)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2(\cos(x)-1)^2}$$

input
```
integrate(sin(x)/(1-cos(x))^3,x, algorithm="maxima")
```

output
```
-1/2/(cos(x) - 1)^2
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{2(\cos(x)-1)^2}$$

input
```
integrate(sin(x)/(1-cos(x))^3,x, algorithm="giac")
```

output
```
-1/2/(cos(x) - 1)^2
```

3.19.9 Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sin(x)}{(1-\cos(x))^3} \, dx = -\frac{1}{2 (\cos(x) - 1)^2}$$

input
```
int(-sin(x)/(cos(x) - 1)^3,x)
```

output
```
-1/(2*(cos(x) - 1)^2)
```

3.20 $\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx$

3.20.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\sin^3(x)}{3(1+\cos(x))^3}$$

output
```
1/3*sin(x)^3/(1+cos(x))^3
```

3.20.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{1}{3}\tan^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^2/(1 + Cos[x])^3,x]
```

output
```
Tan[x/2]^3/3
```

3.20.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(\cos(x)+1)^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3150}$$

$$\frac{\sin^3(x)}{3(\cos(x)+1)^3}$$

input
```
Int[Sin[x]^2/(1 + Cos[x])^3,x]
```

output
```
Sin[x]^3/(3*(1 + Cos[x])^3)
```

3.20.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

3.20.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}$	9
parallelrisch	$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}$	9
risch	$-\frac{2i\left(3\,\mathrm{e}^{2ix}+1\right)}{3\left(\mathrm{e}^{ix}+1\right)^3}$	22
norman	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	37

input
```
int(sin(x)^2/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(1/2*x)^3
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = -\frac{(\cos(x)-1)\sin(x)}{3\left(\cos(x)^2+2\cos(x)+1\right)}$$

input
```
integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="fricas")
```

output
```
-1/3*(cos(x) - 1)*sin(x)/(cos(x)^2 + 2*cos(x) + 1)
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\tan^3\left(\frac{x}{2}\right)}{3}$$

input
```
integrate(sin(x)**2/(1+cos(x))**3,x)
```

output
```
tan(x/2)**3/3
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\sin{(x)}^3}{3\,(\cos{(x)}+1)^3}$$

input
```
integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="maxima")
```

output
```
1/3*sin(x)^3/(cos(x) + 1)^3
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{1}{3}\tan\left(\frac{1}{2}x\right)^3$$

input
```
integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="giac")
```

output
```
1/3*tan(1/2*x)^3
```

3.20.9 Mupad [B] (verification not implemented)

Time = 13.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3}\,dx = \frac{\tan\left(\frac{x}{2}\right)^3}{3}$$

input
```
int(sin(x)^2/(cos(x) + 1)^3,x)
```

output
```
tan(x/2)^3/3
```

3.21 $\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx$

3.21.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx = -\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

output
```
-1/3*sin(x)^3/(1-cos(x))^3
```

3.21.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx = -\frac{1}{3} \cot^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^2/(1 - Cos[x])^3,x]
```

output
```
-1/3*Cot[x/2]^3
```

3.21.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \; 3150$$

$$-\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

input
```
Int[Sin[x]^2/(1 - Cos[x])^3,x]
```

output
```
-1/3*Sin[x]^3/(1 - Cos[x])^3
```

3.21.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

3.21.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{1}{3 \tan \left(\frac{x}{2}\right)^{3}}$	9
parallelrisch	$-\frac{1}{3 \tan \left(\frac{x}{2}\right)^{3}}$	9
risch	$\frac{2 i \left(3 \,{\mathrm e}^{2 i x}+1\right)}{3 \left({\mathrm e}^{i x}-1\right)^{3}}$	22
norman	$\frac{-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}-\frac{2 \left(\tan^{4}\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{3}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2} \tan \left(\frac{x}{2}\right)^{5}}$	43

input
```
int(sin(x)^2/(1-cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/tan(1/2*x)^3
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\, dx = \frac{\cos\left(x\right)^{2} + 2 \, \cos\left(x\right) + 1}{3 \, {\left(\cos\left(x\right) - 1\right)} \sin\left(x\right)}$$

input
```
integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="fricas")
```

output
```
1/3*(cos(x)^2 + 2*cos(x) + 1)/((cos(x) - 1)*sin(x))
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\, dx = - \frac{1}{3 \tan^{3}{\left(\frac{x}{2} \right)}}$$

input
```
integrate(sin(x)**2/(1-cos(x))**3,x)
```

output
```
-1/(3*tan(x/2)**3)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{(\cos(x)+1)^3}{3\sin(x)^3}$$

input
```
integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="maxima")
```

output
```
-1/3*(cos(x) + 1)^3/sin(x)^3
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{1}{3\tan\left(\frac{1}{2}x\right)^3}$$

input
```
integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="giac")
```

output
```
-1/3/tan(1/2*x)^3
```

3.21.9 Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{\cot\left(\frac{x}{2}\right)^3}{3}$$

input
```
int(-sin(x)^2/(cos(x) - 1)^3,x)
```

output
```
-cot(x/2)^3/3
```

3.22 $\int \frac{\sin^3(x)}{(1+\cos(x))^3} dx$

3.22.1 Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3} dx = \frac{2}{1+\cos(x)} + \log(1+\cos(x))$$

output
```
2/(1+cos(x))+ln(1+cos(x))
```

3.22.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3} dx = 2\log\left(\cos\left(\frac{x}{2}\right)\right) + \tan^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sin[x]^3/(1 + Cos[x])^3,x]
```

output
```
2*Log[Cos[x/2]] + Tan[x/2]^2
```

3.22.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(\cos(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1-\cos(x)}{(\cos(x)+1)^2}d\cos(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{(\cos(x)+1)^2}+\frac{1}{-\cos(x)-1}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

input
```
Int[Sin[x]^3/(1 + Cos[x])^3,x]
```

output
```
2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

3.22.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.22.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$	15
default	$\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$	15
parallelrisch	$\tan^{2}\left(\frac{x}{2}\right)-\ln\left(\frac{2}{\cos(x)+1}\right)$	19
risch	$-ix+\frac{4\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+1\right)^{2}}+2\ln\left(\mathrm{e}^{ix}+1\right)$	32
norman	$\frac{\tan^{8}\left(\frac{x}{2}\right)-8\left(\tan^{2}\left(\frac{x}{2}\right)\right)-6\left(\tan^{4}\left(\frac{x}{2}\right)\right)-3}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}-\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)$	48

input
```
int(sin(x)^3/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
```

output
```
2/(cos(x)+1)+ln(cos(x)+1)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{\cos(x)+1}$$

input
```
integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="fricas")
```

output
```
((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 9.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2\log(\cos(x)+1)\cos^2(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{4\log(\cos(x)+1)\cos(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{2\log(\cos(x)+1)}{2\cos^2(x)+4\cos(x)+2} + \frac{\sin^2(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{2\cos(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{2}{2\cos^2(x)+4\cos(x)+2}$$

input
```
integrate(sin(x)**3/(1+cos(x))**3,x)
```

output
```
2*log(cos(x) + 1)*cos(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 4*log(cos(x) +
1)*cos(x)/(2*cos(x)**2 + 4*cos(x) + 2) + 2*log(cos(x) + 1)/(2*cos(x)**2 +
4*cos(x) + 2) + sin(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 2*cos(x)/(2*cos(x
)**2 + 4*cos(x) + 2) + 2/(2*cos(x)**2 + 4*cos(x) + 2)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="maxima")
```

output
```
2/(cos(x) + 1) + log(cos(x) + 1)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="giac")
```

output
```
2/(cos(x) + 1) + log(cos(x) + 1)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx = \ln(\cos(x)+1) + \frac{2}{\cos(x)+1}$$

input
```
int(sin(x)^3/(cos(x) + 1)^3,x)
```

output
```
log(cos(x) + 1) + 2/(cos(x) + 1)
```

3.23 $\int \frac{\sin^3(x)}{(1-\cos(x))^3} \, dx$

3.23.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3} \, dx = -\frac{2}{1-\cos(x)} - \log(1-\cos(x))$$

output
```
-2/(1-cos(x))-ln(1-cos(x))
```

3.23.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3} \, dx = -\cot^2\left(\frac{x}{2}\right) - 2\log\left(\cos\left(\frac{x}{2}\right)\right) - 2\log\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]^3/(1 - Cos[x])^3,x]
```

output
```
-Cot[x/2]^2 - 2*Log[Cos[x/2]] - 2*Log[Tan[x/2]]
```

3.23.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{\cos(x)+1}{(1-\cos(x))^2}d(-\cos(x))$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{2}{(1-\cos(x))^2}+\frac{1}{\cos(x)-1}\right)d(-\cos(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

input
```
Int[Sin[x]^3/(1 - Cos[x])^3,x]
```

output
```
-2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

3.23.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.23.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2}{\cos(x)-1}-\ln\left(\cos\left(x\right)-1\right)$	17
default	$\frac{2}{\cos(x)-1}-\ln\left(\cos\left(x\right)-1\right)$	17
parallelrisch	$-2\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\frac{2}{\cos(x)+1}\right)-\left(\cot^{2}\left(\frac{x}{2}\right)\right)$	26
risch	$ix+\frac{4\,\mathrm{e}^{ix}}{(\mathrm{e}^{ix}-1)^{2}}-2\ln\left(\mathrm{e}^{ix}-1\right)$	32
norman	$\frac{-3\left(\tan^{5}\left(\frac{x}{2}\right)\right)-3\left(\tan^{7}\left(\frac{x}{2}\right)\right)-\left(\tan^{9}\left(\frac{x}{2}\right)\right)-\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}\tan\left(\frac{x}{2}\right)^{5}}-2\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)$	68

input
```
int(sin(x)^3/(1-cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(cos(x)-1)-ln(cos(x)-1)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int\frac{\sin^{3}(x)}{(1-\cos(x))^{3}}\,dx=-\frac{\left(\cos\left(x\right)-1\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)-2}{\cos\left(x\right)-1}$$

input
```
integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="fricas")
```

output
```
-((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = -\frac{2\log(\cos(x)-1)\cos^2(x)}{2\cos^2(x)-4\cos(x)+2} + \frac{4\log(\cos(x)-1)\cos(x)}{2\cos^2(x)-4\cos(x)+2} - \frac{2\log(\cos(x)-1)}{2\cos^2(x)-4\cos(x)+2} - \frac{\sin^2(x)}{2\cos^2(x)-4\cos(x)+2} + \frac{2\cos(x)}{2\cos^2(x)-4\cos(x)+2} - \frac{2}{2\cos^2(x)-4\cos(x)+2}$$

input
```
integrate(sin(x)**3/(1-cos(x))**3,x)
```

output
```
-2*log(cos(x) - 1)*cos(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 4*log(cos(x) -
 1)*cos(x)/(2*cos(x)**2 - 4*cos(x) + 2) - 2*log(cos(x) - 1)/(2*cos(x)**2 -
 4*cos(x) + 2) - sin(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 2*cos(x)/(2*cos(
x)**2 - 4*cos(x) + 2) - 2/(2*cos(x)**2 - 4*cos(x) + 2)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \log(\cos(x)-1)$$

input
```
integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="maxima")
```

output
```
2/(cos(x) - 1) - log(cos(x) - 1)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \log(-\cos(x)+1)$$

input
```
integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="giac")
```

output
```
2/(cos(x) - 1) - log(-cos(x) + 1)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(x)}{(1-\cos(x))^3}\,dx = \frac{2}{\cos(x)-1} - \ln(\cos(x)-1)$$

input
```
int(-sin(x)^3/(cos(x) - 1)^3,x)
```

output
```
2/(cos(x) - 1) - log(cos(x) - 1)
```

3.24 $\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$

3.24.1 Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx=-\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3}-\frac{\sin^3(x)}{3b}$$

output
```
-1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctan((a-b)^(1/2)*ta
n(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cos(x))*sin(x)/b^3-1/3*sin(
x)^3/b
```

3.24.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx=\frac{-12a^3x+18ab^2x-24(-a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+3b(4a^2-5b^2)\sin(x)-3ab^2\sin(2x)+b^3\sin(}{12b^4}$$

input
```
Integrate[Sin[x]^4/(a + b*Cos[x]),x]
```

output
```
(-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTanh[((a - b)*Tan[x/2])
/Sqrt[-a^2 + b^2]] + 3*b*(4*a^2 - 5*b^2)*Sin[x] - 3*a*b^2*Sin[2*x] + b^3*S
in[3*x])/(12*b^4)
```

3.24.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3042, 3174, 25, 3042, 3344, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^4}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{\int -\frac{(b+a\cos(x))\sin^2(x)}{a+b\cos(x)}dx}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(b+a\cos(x))\sin^2(x)}{a+b\cos(x)}dx}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2\left(b+a\sin\left(x+\frac{\pi}{2}\right)\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3344}$$

$$\frac{\frac{\int -\frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cos(x)}{a+b\cos(x)}dx}{2b^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{2b^2}-\frac{\int \frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cos(x)}{a+b\cos(x)}dx}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\int\frac{b(a^2-2b^2)+a(2a^2-3b^2)\sin\left(x+\frac{\pi}{2}\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{2(a^2-b^2)^2\int\frac{1}{a+b\cos(x)}dx}{b}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{2(a^2-b^2)^2\int\frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{4(a^2-b^2)^2\int\frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{b}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\sin(x)(2(a^2-b^2)-ab\cos(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{4(a^2-b^2)^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{2b^2}}{b}-\frac{\sin^3(x)}{3b}$$

input
```
Int[Sin[x]^4/(a + b*Cos[x]),x]
```

output
```
-1/3*Sin[x]^3/b + (-1/2*((a*(2*a^2 - 3*b^2)*x)/b - (4*(a^2 - b^2)^2*ArcTan
[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b^2 + (
(2*(a^2 - b^2) - a*b*Cos[x])*Sin[x])/(2*b^2))/b
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

3.24. $\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$

3.24.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2\left(\frac{\left(-a^{2}b-\frac{1}{2}a\,b^{2}+b^{3}\right)\left(\tan^{5}\left(\frac{x}{2}\right)\right)+\left(-2a^{2}b+\frac{10}{3}b^{3}\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)+\left(-a^{2}b+b^{3}+\frac{1}{2}a\,b^{2}\right)\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}+\frac{a\left(2a^{2}-3b^{2}\right)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{2}\right)}{b^{4}}+\frac{2(a+b)^{2}(}{}$
risch	$-\frac{a^{3}x}{b^{4}}+\frac{3ax}{2b^{2}}-\frac{i\mathrm{e}^{ix}a^{2}}{2b^{3}}+\frac{5i\mathrm{e}^{ix}}{8b}+\frac{i\mathrm{e}^{-ix}a^{2}}{2b^{3}}-\frac{5i\mathrm{e}^{-ix}}{8b}+\frac{\sqrt{-a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{ix}-\frac{i\sqrt{-a^{2}+b^{2}}-a}{b}\right)a^{2}}{b^{4}}-\frac{\sqrt{-a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{ix}-\frac{i\sqrt{-a^{2}}}{}\right.}{b^{2}}$

input
```
int(sin(x)^4/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
-2/b^4*(((-a^2*b-1/2*a*b^2+b^3)*tan(1/2*x)^5+(-2*a^2*b+10/3*b^3)*tan(1/2*x
)^3+(-a^2*b+b^3+1/2*a*b^2)*tan(1/2*x))/(1+tan(1/2*x)^2)^3+1/2*a*(2*a^2-3*b
^2)*arctan(tan(1/2*x)))+2*(a+b)^2*(a-b)^2/b^4/((a-b)*(a+b))^(1/2)*arctan((
a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.34

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$$
$$=\left[-\frac{3\,(a^2-b^2)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)+3\,(2\,a^3-3\,ab^2)x}{6\,b^4}\right.$$

input
```
integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[-1/6*(3*(a^2 - b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*co
s(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(
x)^2 + 2*a*b*cos(x) + a^2)) + 3*(2*a^3 - 3*a*b^2)*x - (2*b^3*cos(x)^2 - 3*
a*b^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4, 1/6*(6*(a^2 - b^2)^(3/2)*arct
an(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - 3*(2*a^3 - 3*a*b^2)*x + (2*
b^3*cos(x)^2 - 3*a*b^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4]
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**4/(a+b*cos(x)),x)
```

output
```
Timed out
```

3.24.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.87

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = -\frac{(2\,a^3-3\,ab^2)x}{2\,b^4} - \frac{2\,(a^4-2\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}b^4} + \frac{6\,a^2\tan\left(\frac{1}{2}\,x\right)^5+3\,ab\tan\left(\frac{1}{2}\,x\right)^5-6\,b^2\tan\left(\frac{1}{2}\,x\right)^5+12\,a^2\tan\left(\frac{1}{2}\,x\right)^3-20\,b^2\tan\left(\frac{1}{2}\,x\right)^3+6\,a^2\tan\left(\frac{1}{2}\,x\right) -}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3b^3}$$

input
```
integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-1/2*(2*a^3 - 3*a*b^2)*x/b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*x/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x))/sqrt(a^2
- b^2)))/(sqrt(a^2 - b^2)*b^4) + 1/3*(6*a^2*tan(1/2*x)^5 + 3*a*b*tan(1/2*x
)^5 - 6*b^2*tan(1/2*x)^5 + 12*a^2*tan(1/2*x)^3 - 20*b^2*tan(1/2*x)^3 + 6*a
^2*tan(1/2*x) - 3*a*b*tan(1/2*x) - 6*b^2*tan(1/2*x))/((tan(1/2*x)^2 + 1)^3
*b^3)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 1677, normalized size of antiderivative = 16.12

$$\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx = \text{Too large to display}$$

input
```
int(sin(x)^4/(a + b*cos(x)),x)
```

output
```
((4*tan(x/2)^3*(3*a^2 - 5*b^2))/(3*b^3) - (tan(x/2)*(a*b - 2*a^2 + 2*b^2))
/b^3 + (tan(x/2)^5*(a*b + 2*a^2 - 2*b^2))/b^3)/(3*tan(x/2)^2 + 3*tan(x/2)^
4 + tan(x/2)^6 + 1) - (2*atanh((64*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4
*b^2)^(1/2))/(128*a*b^2 + 112*a^2*b - 352*a^3 - 64*b^3 + (16*a^4)/b + (320
*a^5)/b^2 - (112*a^6)/b^3 - (96*a^7)/b^4 + (48*a^8)/b^5) + (144*a^2*tan(x/
2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^4 + 16*a^4*b + 320*
a^5 - 64*b^5 + 112*a^2*b^3 - 352*a^3*b^2 - (112*a^6)/b - (96*a^7)/b^2 + (4
8*a^8)/b^3) + (80*a^3*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/
(128*a*b^5 + 320*a^5*b - 112*a^6 - 64*b^6 + 112*a^2*b^4 - 352*a^3*b^3 + 16
*a^4*b^2 - (96*a^7)/b + (48*a^8)/b^2) - (144*a^4*tan(x/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^6 - 112*a^6*b - 96*a^7 - 64*b^7 + 112*
a^2*b^5 - 352*a^3*b^4 + 16*a^4*b^3 + 320*a^5*b^2 + (48*a^8)/b) + (48*a^5*t
an(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^7 - 96*a^7*b +
 48*a^8 - 64*b^8 + 112*a^2*b^6 - 352*a^3*b^5 + 16*a^4*b^4 + 320*a^5*b^3 -
112*a^6*b^2) - (192*a*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/
(128*a*b^3 - 352*a^3*b + 16*a^4 - 64*b^4 + 112*a^2*b^2 + (320*a^5)/b - (11
2*a^6)/b^2 - (96*a^7)/b^3 + (48*a^8)/b^4))*(-(a + b)^3*(a - b)^3)^(1/2))/b
^4 + (a*atan(((a*(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9
- 4*b^9 + 7*a^2*b^7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 1
6*a^7*b^2))/b^6 - (a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 ...
```

3.25 $\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx$

3.25.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = -\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

output
```
-a*cos(x)/b^2+1/2*cos(x)^2/b+(a^2-b^2)*ln(a+b*cos(x))/b^3
```

3.25.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = -\frac{a\cos(x)}{b^2}+\frac{\cos(2x)}{4b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

input
```
Integrate[Sin[x]^3/(a + b*Cos[x]),x]
```

output
```
-((a*Cos[x])/b^2) + Cos[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cos[x]])/b^3
```

3.25.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{b^2-b^2\cos^2(x)}{a+b\cos(x)}d(b\cos(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(a-b\cos(x)+\frac{b^2-a^2}{a+b\cos(x)}\right)d(b\cos(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\left(a^2-b^2\right)\log(a+b\cos(x))+ab\cos(x)-\frac{1}{2}b^2\cos^2(x)}{b^3}$$

input
```
Int[Sin[x]^3/(a + b*Cos[x]),x]
```

output
```
-((a*b*Cos[x] - (b^2*Cos[x]^2)/2 - (a^2 - b^2)*Log[a + b*Cos[x]])/b^3)
```

3.25.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.25.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{-\frac{b\left(\cos^2(x)\right)}{2}+\cos(x)a}{b^2}+\frac{(a^2-b^2)\ln(a+\cos(x)b)}{b^3}$
default	$-\frac{-\frac{b\left(\cos^2(x)\right)}{2}+\cos(x)a}{b^2}+\frac{(a^2-b^2)\ln(a+\cos(x)b)}{b^3}$
parallelrisch	$\frac{(a^2-b^2)\ln\left(\frac{a+\cos(x)b}{\cos(x)+1}\right)+(-a^2+b^2)\ln\left(\frac{1}{\cos(x)+1}\right)-b\left(\cos(x)a-\frac{b\cos(2x)}{4}+a+\frac{b}{4}\right)}{b^3}$
norman	$\frac{\frac{2a\left(\tan^4\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2a-2b}{3b^2}+\frac{(4a+2b)\left(\tan^6\left(\frac{x}{2}\right)\right)}{3b^2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}+\frac{(a-b)(a+b)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{b^3}-\frac{(a-b)(a+b)\ln(1+\text{ta}}{b^3}$
risch	$-\frac{ia^2x}{b^3}+\frac{ix}{b}+\frac{e^{2ix}}{8b}-\frac{a\,e^{ix}}{2b^2}-\frac{a\,e^{-ix}}{2b^2}+\frac{e^{-2ix}}{8b}+\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)a^2}{b^3}-\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{b}$

input
```
int(sin(x)^3/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*cos(x)^2+cos(x)*a)+(a^2-b^2)*ln(a+cos(x)*b)/b^3
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx=\frac{b^2\cos{(x)}^2-2\,ab\cos{(x)}+2\,(a^2-b^2)\log{(-b\cos{(x)}-a)}}{2\,b^3}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
1/2*(b^2*cos(x)^2 - 2*a*b*cos(x) + 2*(a^2 - b^2)*log(-b*cos(x) - a))/b^3
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(34) = 68$.

Time = 173.47 (sec) , antiderivative size = 1421, normalized size of antiderivative = 35.52

$$\int \frac{\sin^3(x)}{a + b\cos(x)}\, dx = \text{Too large to display}$$

input
```
integrate(sin(x)**3/(a+b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**
2 + 1) - 2*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1)
 - log(tan(x/2) - 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)
*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*log(tan(x/2) + 1)*tan(x
/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)/(tan(x/2)**4
+ 2*tan(x/2)**2 + 1) + log(tan(x/2)**2 + 1)*tan(x/2)**4/(tan(x/2)**4 + 2*t
an(x/2)**2 + 1) + 2*log(tan(x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(
x/2)**2 + 1) + log(tan(x/2)**2 + 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*
tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (-4*
tan(x/2)**2/(b*tan(x/2)**4 + 2*b*tan(x/2)**2 + b) - 2/(b*tan(x/2)**4 + 2*b
*tan(x/2)**2 + b), Eq(a, b)), ((-sin(x)**2*cos(x) - 2*cos(x)**3/3)/a, Eq(b
, 0)), (a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**4/(b*
*3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + 2*a**2*log(-sqrt(-a/(a - b)
- b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**
2 + b**3) + a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**3*tan(x
/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a - b)
) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) +
 2*a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**3*tan
(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a -
b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - a**2*l...
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b\cos(x)^2 - 2\,a\cos(x)}{2\,b^2} + \frac{(a^2-b^2)\log(b\cos(x)+a)}{b^3}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(b*cos(x) + a)/b^3
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{b\cos(x)^2 - 2\,a\cos(x)}{2\,b^2} + \frac{(a^2-b^2)\log(|b\cos(x)+a|)}{b^3}$$

input
```
integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

output
```
1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(abs(b*cos(x) + a))/b^3
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx = \frac{\cos(x)^2}{2\,b} + \frac{\ln(a+b\,\cos(x))\,(a^2-b^2)}{b^3} - \frac{a\,\cos(x)}{b^2}$$

input
```
int(sin(x)^3/(a + b*cos(x)),x)
```

output
```
cos(x)^2/(2*b) + (log(a + b*cos(x))*(a^2 - b^2))/b^3 - (a*cos(x))/b^2
```

3.26 $\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx$

3.26.1 Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx=\frac{ax}{b^2}-\frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}-\frac{\sin(x)}{b}$$

output
```
a*x/b^2-sin(x)/b-2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*
(a+b)^(1/2)/b^2
```

3.26.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx=\frac{ax-2\sqrt{-a^2+b^2}\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)-b\sin(x)}{b^2}$$

input
```
Integrate[Sin[x]^2/(a + b*Cos[x]),x]
```

output
```
(a*x - 2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]] - b
*Sin[x])/b^2
```

3.26.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 3174, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{\int -\frac{b+a\cos(x)}{a+b\cos(x)}dx}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b+a\cos(x)}{a+b\cos(x)}dx}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\sin\left(x+\frac{\pi}{2}\right)}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{ax}{b}-\frac{\left(a^2-b^2\right)\int \frac{1}{a+b\cos(x)}dx}{b}}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{ax}{b}-\frac{\left(a^2-b^2\right)\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{ax}{b}-\frac{2\left(a^2-b^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{b}}{b}-\frac{\sin(x)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{ax}{b}-\frac{2(a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{\sin(x)}{b}$$

input
```
Int[Sin[x]^2/(a + b*Cos[x]),x]
```

output
```
((a*x)/b - (2*(a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b]))/b - Sin[x]/b
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{-\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}+2a\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2(a+b)(a-b)\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}$	78
risch	$\frac{ax}{b^2}+\frac{i e^{ix}}{2b}-\frac{i e^{-ix}}{2b}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}-\frac{i\sqrt{-a^2+b^2}-a}{b}\right)}{b^2}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{b^2}$	118

input
```
int(sin(x)^2/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(-b*tan(1/2*x)/(1+tan(1/2*x)^2)+a*arctan(tan(1/2*x)))-2*(a+b)*(a-b)/
b^2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.61

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \left[\frac{2\,ax-2\,b\sin(x)+\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,b^2}, \frac{ax-b\sin(\right.$$

input
```
integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x - 2*b*sin(x) + sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b
^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^
2*cos(x)^2 + 2*a*b*cos(x) + a^2)))/b^2, (a*x - b*sin(x) - sqrt(a^2 - b^2)*
arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))))/b^2]
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(49) = 98$.

Time = 60.78 (sec) , antiderivative size = 991, normalized size of antiderivative = 16.80

$$\int \frac{\sin^2(x)}{a + b\cos(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(x)**2/(a+b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - log(tan
(x/2) - 1)/(tan(x/2)**2 + 1) + log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2
+ 1) + log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - 2*tan(x/2)/(tan(x/2)**2 + 1))
, Eq(a, 0) & Eq(b, 0)), (x*tan(x/2)**2/(b*tan(x/2)**2 + b) + x/(b*tan(x/2)
**2 + b) - 2*tan(x/2)/(b*tan(x/2)**2 + b), Eq(a, b)), (-x*tan(x/2)**2/(b*t
an(x/2)**2 + b) - x/(b*tan(x/2)**2 + b) - 2*tan(x/2)/(b*tan(x/2)**2 + b),
Eq(a, -b)), ((x*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2)/a, Eq(b, 0)
), (a*x*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b
/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) + a*x*sqrt(-a/(
a - b) - b/(a - b))/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*
sqrt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(
x/2))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sq
rt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/
2))/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b)
- b/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2
/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b
/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqrt(-a/
(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*
b*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)/(b**2*sqrt(-a/(a - b) - b/(a - b))
*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) - b*log(-sqrt(-a/(a -...
```

3.26.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{a + b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{ax}{b^2} + \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)b}$$

input
```
integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

output
```
a*x/b^2 + 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*x) - b*tan(1/2*x))/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 - 2*tan(1/2*x)/(
(tan(1/2*x)^2 + 1)*b)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

$$\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}}{a\cos\left(\frac{x}{2}\right)+b\cos\left(\frac{x}{2}\right)}\right)\sqrt{b^2-a^2}}{b^2} - \frac{\sin(x)}{b} + \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b^2}$$

input
```
int(sin(x)^2/(a + b*cos(x)),x)
```

output
```
(2*atanh((sin(x/2)*(b^2 - a^2)^(1/2))/(a*cos(x/2) + b*cos(x/2)))*(b^2 - a^
2)^(1/2))/b^2 - sin(x)/b + (2*a*atan(sin(x/2)/cos(x/2)))/b^2
```

3.27 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$

3.27.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(a+b\cos(x))}{b}$$

output
```
-ln(a+b*cos(x))/b
```

3.27.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(a+b\cos(x))}{b}$$

input
```
Integrate[Sin[x]/(a + b*Cos[x]),x]
```

output
```
-(Log[a + b*Cos[x]]/b)
```

3.27.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$-\frac{\int \frac{1}{a+b\cos(x)}d(b\cos(x))}{b}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{\log(a+b\cos(x))}{b}$$

input
```
Int[Sin[x]/(a + b*Cos[x]),x]
```

output
```
-(Log[a + b*Cos[x]]/b)
```

3.27.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.27.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+\cos(x)b)}{b}$	13
default	$-\frac{\ln(a+\cos(x)b)}{b}$	13
parallelrisch	$\frac{\ln\left(\frac{1}{\cos(x)+1}\right)-\ln\left(\frac{a+\cos(x)b}{\cos(x)+1}\right)}{b}$	29
risch	$\frac{ix}{b}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2a\,\mathrm{e}^{ix}}{b}+1\right)}{b}$	33
norman	$\frac{\ln(1+\tan^2(\frac{x}{2}))}{b}-\frac{\ln(a(\tan^2(\frac{x}{2}))-b(\tan^2(\frac{x}{2}))+a+b)}{b}$	41

input
```
int(sin(x)/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+cos(x)*b)/b
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
-log(-b*cos(x) - a)/b
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = \begin{cases} -\frac{\log\left(\frac{a}{b}+\cos(x)\right)}{b} & \text{for } b\neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x)
```

output
```
Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log\left(b\cos(x)+a\right)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
-log(b*cos(x) + a)/b
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log\left(|b\cos(x)+a|\right)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-log(abs(b*cos(x) + a))/b
```

3.27.9 Mupad [B] (verification not implemented)

Time = 13.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\ln\left(a+b\ \cos\left(x\right)\right)}{b}$$

input
```
int(sin(x)/(a + b*cos(x)),x)
```

output
```
-log(a + b*cos(x))/b
```

3.28 $\int \frac{1}{a+b\cos(x)}\,dx$

3.28.1 Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \frac{1}{a+b\cos(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

output
```
2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cos(x)}\,dx=-\frac{2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

input
```
Integrate[(a + b*Cos[x])^(-1),x]
```

output
```
(-2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

3.28.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3138$$

$$2\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cos[x])^(-1),x]
```

output
```
(2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

3.28.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.28.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}$	36
risch	$-\frac{\ln\left(e^{ix}+\frac{i a^2-i b^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^{ix}-\frac{i a^2-i b^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	125

input
```
int(1/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.26

$$\int \frac{1}{a+b\cos(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,(a^2-b^2)}, \frac{\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*co
s(x) + a^2))/(a^2 - b^2), arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))
/sqrt(a^2 - b^2)]
```

3.28.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(34) = 68$.

Time = 1.71 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.43

$$\int \frac{1}{a+b\cos(x)}\,dx = \begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)\right) & \text{for } a=0 \wedge b=0 \\ \frac{\tan\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ \frac{1}{b\tan\left(\frac{x}{2}\right)} & \text{for } a=-b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cos(x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b,
0)), (tan(x/2)/b, Eq(a, b)), (1/(b*tan(x/2)), Eq(a, -b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x/2))/(a*sqrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a
/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(a*s
qrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a/(a - b) - b/(a - b))), True))
```

3.28.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{1}{a+b\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

input
```
integrate(1/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x))/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\cos(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(2a-2b)}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b*cos(x)),x)
```

output
```
(2*atan((tan(x/2)*(2*a - 2*b))/(2*(a^2 - b^2)^(1/2))))/(a^2 - b^2)^(1/2)
```

3.29 $\int \frac{\csc(x)}{a+b\cos(x)}\,dx$

3.29.1 Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx=\frac{\log(1-\cos(x))}{2(a+b)}-\frac{\log(1+\cos(x))}{2(a-b)}+\frac{b\log(a+b\cos(x))}{a^2-b^2}$$

output
```
1/2*ln(1-cos(x))/(a+b)-1/2*ln(1+cos(x))/(a-b)+b*ln(a+b*cos(x))/(a^2-b^2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx=\frac{(a-b)\log(1-\cos(x))-(a+b)\log(1+\cos(x))+2b\log(a+b\cos(x))}{2(a-b)(a+b)}$$

input
```
Integrate[Csc[x]/(a + b*Cos[x]),x]
```

output
```
((a - b)*Log[1 - Cos[x]] - (a + b)*Log[1 + Cos[x]] + 2*b*Log[a + b*Cos[x]]
)/(2*(a - b)*(a + b))
```

3.29.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3147$$

$$-b\int \frac{1}{(a+b\cos(x))\left(b^2-b^2\cos^2(x)\right)}d(b\cos(x))$$

$$\Big\downarrow \ 477$$

$$-\frac{\int \left(-\frac{b^2}{(a^2-b^2)(a+b\cos(x))}+\frac{b}{2(a+b)(b-b\cos(x))}+\frac{b}{2(a-b)(\cos(x)b+b)}\right)d(b\cos(x))}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{b^2\log(a+b\cos(x))}{a^2-b^2}-\frac{b\log(b-b\cos(x))}{2(a+b)}+\frac{b\log(b\cos(x)+b)}{2(a-b)}}{b}$$

input
```
Int[Csc[x]/(a + b*Cos[x]),x]
```

output
```
-((-1/2*(b*Log[b - b*Cos[x]])/(a + b) - (b^2*Log[a + b*Cos[x]])/(a^2 - b^2
) + (b*Log[b + b*Cos[x]])/(2*(a - b)))/b)
```

3.29.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.29.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{a^2-b^2}$	47
parallelrisch	$\frac{b\ln\left(\frac{2\cos(x)b+2a}{\cos(x)+1}\right)+\ln(\csc(x)-\cot(x))(a-b)}{a^2-b^2}$	47
default	$-\frac{\ln(\cos(x)+1)}{2a-2b}+\frac{\ln(\cos(x)-1)}{2a+2b}+\frac{b\ln(a+\cos(x)b)}{(a-b)(a+b)}$	54
risch	$-\frac{ix}{a+b}+\frac{ix}{a-b}-\frac{2ixb}{a^2-b^2}+\frac{\ln(e^{ix}-1)}{a+b}-\frac{\ln(e^{ix}+1)}{a-b}+\frac{b\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{a^2-b^2}$	101

input
```
int(csc(x)/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)*ln(tan(1/2*x))+1/(a^2-b^2)*b*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+a+b)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{2\,b\log\left(-b\cos\left(x\right)-a\right)-(a+b)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+(a-b)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)}$$

input
```
integrate(csc(x)/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
1/2*(2*b*log(-b*cos(x) - a) - (a + b)*log(1/2*cos(x) + 1/2) + (a - b)*log(
-1/2*cos(x) + 1/2))/(a^2 - b^2)
```

3.29.6 Sympy [F]

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \int \frac{\csc\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

input
```
integrate(csc(x)/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)/(a + b*cos(x)), x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{b\log\left(b\cos\left(x\right)+a\right)}{a^2-b^2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a-b\right)}+\frac{\log\left(\cos\left(x\right)-1\right)}{2\left(a+b\right)}$$

input
```
integrate(csc(x)/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
b*log(b*cos(x) + a)/(a^2 - b^2) - 1/2*log(cos(x) + 1)/(a - b) + 1/2*log(co
s(x) - 1)/(a + b)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{b^2\log(|b\cos(x)+a|)}{a^2b-b^3} - \frac{\log(\cos(x)+1)}{2(a-b)} + \frac{\log(-\cos(x)+1)}{2(a+b)}$$

input
```
integrate(csc(x)/(a+b*cos(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(b*cos(x) + a))/(a^2*b - b^3) - 1/2*log(cos(x) + 1)/(a - b) + 1
/2*log(-cos(x) + 1)/(a + b)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\csc(x)}{a+b\cos(x)}\,dx = \frac{\ln(\cos(x)-1)}{2(a+b)} - \frac{\ln(\cos(x)+1)}{2(a-b)} + \frac{b\ln(a+b\cos(x))}{a^2-b^2}$$

input
```
int(1/(sin(x)*(a + b*cos(x))),x)
```

output
```
log(cos(x) - 1)/(2*(a + b)) - log(cos(x) + 1)/(2*(a - b)) + (b*log(a + b*c
os(x)))/(a^2 - b^2)
```

3.30 $\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$

3.30.1 Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = -\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

output
```
-2*b^2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+
(b-a*cos(x))*csc(x)/(a^2-b^2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = -\frac{2b^2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

input
```
Integrate[Csc[x]^2/(a + b*Cos[x]),x]
```

output
```
(-2*b^2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) +
 ((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

3.30.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3175, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^2\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{\int \frac{b^2}{a+b\cos(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{b^2\int \frac{1}{a+b\cos(x)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{b^2\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{2b^2\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}$$

input
```
Int[Csc[x]^2/(a + b*Cos[x]),x]
```

output
```
(-2*b^2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a +
b]*(a^2 - b^2)) + ((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

3.30.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)}{2a-2b}-\frac{1}{2(a+b)\tan\left(\frac{x}{2}\right)}-\frac{2b^2\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$	78
risch	$-\frac{2i(-\mathrm{e}^{ix}b+a)}{(\mathrm{e}^{2ix}-1)(a^2-b^2)}+\frac{b^2\ln\left(\mathrm{e}^{ix}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}-\frac{b^2\ln\left(\mathrm{e}^{ix}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	186

input
```
int(csc(x)^2/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(1/2*x)/(a-b)-1/2/(a+b)/tan(1/2*x)-2/(a-b)/(a+b)*b^2/((a-b)*(a+b))^
(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.43

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$$
$$= \left[\frac{\sqrt{-a^2+b^2}b^2\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)\sin(x)+2\,a^2b-2\,b^3-2\,(a^3-}{2\,(a^4-2\,a^2b^2+b^4)\sin(x)}\right.$$
$$\left.-\frac{\sqrt{a^2-b^2}b^2\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)\sin(x)-a^2b+b^3+(a^3-ab^2)\cos(x)}{(a^4-2\,a^2b^2+b^4)\sin(x)}\right]$$

input
```
integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a^2 + b^2)*b^2*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*
sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*
b*cos(x) + a^2))*sin(x) + 2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x))/((a^4
- 2*a^2*b^2 + b^4)*sin(x)), -(sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(x) + b)/(
sqrt(a^2 - b^2)*sin(x)))*sin(x) - a^2*b + b^3 + (a^3 - a*b^2)*cos(x))/((a^
4 - 2*a^2*b^2 + b^4)*sin(x))]
```

3.30.6 Sympy [F]

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$$

input
```
integrate(csc(x)**2/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)**2/(a + b*cos(x)), x)
```

3.30.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^2}{(a^2-b^2)^{\frac{3}{2}}} + \frac{\tan\left(\frac{1}{2}x\right)}{2(a-b)} - \frac{1}{2(a+b)\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + 1/2*tan(1/2*x)/(a - b)
 - 1/2/((a + b)*tan(1/2*x))
```

3.30.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2a-2b} - \frac{2b^2\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(a^2-b^2)}{(a+b)^{3/2}\sqrt{a-b}}\right)}{(a+b)^{3/2}(a-b)^{3/2}} - \frac{a-b}{\tan\left(\frac{x}{2}\right)(a+b)(2a-2b)}$$

input
```
int(1/(sin(x)^2*(a + b*cos(x))),x)
```

output
```
tan(x/2)/(2*a - 2*b) - (2*b^2*atan((tan(x/2)*(a^2 - b^2))/((a + b)^(3/2)*(
a - b)^(1/2))))/((a + b)^(3/2)*(a - b)^(3/2)) - (a - b)/(tan(x/2)*(a + b)*
(2*a - 2*b))
```

3.30. $\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$

3.31 $\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx$

3.31.1 Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx=\frac{(b-a\cos(x))\csc^2(x)}{2(a^2-b^2)}+\frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2}-\frac{(a-2b)\log(1+\cos(x))}{4(a-b)^2}-\frac{b^3\log(a+b\cos(x))}{(a^2-b^2)^2}$$

output
```
1/2*(b-a*cos(x))*csc(x)^2/(a^2-b^2)+1/4*(a+2*b)*ln(1-cos(x))/(a+b)^2-1/4*(
a-2*b)*ln(1+cos(x))/(a-b)^2-b^3*ln(a+b*cos(x))/(a^2-b^2)^2
```

3.31.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx=\frac{1}{8}\left(-\frac{\csc^2\left(\frac{x}{2}\right)}{a+b}-\frac{4(a-2b)\log\left(\cos\left(\frac{x}{2}\right)\right)}{(a-b)^2}-\frac{8b^3\log(a+b\cos(x))}{(a^2-b^2)^2}+\frac{4(a+2b)\log\left(\sin\left(\frac{x}{2}\right)\right)}{(a+b)^2}+\frac{\sec^2\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Csc[x]^3/(a + b*Cos[x]),x]
```

output
```
(-(Csc[x/2]^2/(a + b)) - (4*(a - 2*b)*Log[Cos[x/2]])/(a - b)^2 - (8*b^3*Lo
g[a + b*Cos[x]])/(a^2 - b^2)^2 + (4*(a + 2*b)*Log[Sin[x/2]])/(a + b)^2 + S
ec[x/2]^2/(a - b))/8
```

3.31.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^3\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3147}$$

$$-b^3\int \frac{1}{(a+b\cos(x))\left(b^2-b^2\cos^2(x)\right)^2}d(b\cos(x))$$

$$\Big\downarrow \text{ 477}$$

$$-\frac{\int \left(\frac{b^4}{(a^2-b^2)^2(a+b\cos(x))}+\frac{b^2}{4(a+b)(b-b\cos(x))^2}+\frac{b^2}{4(a-b)(\cos(x)b+b)^2}+\frac{(a+2b)b}{4(a+b)^2(b-b\cos(x))}+\frac{(a-2b)b}{4(a-b)^2(\cos(x)b+b)}\right)d(b\cos(x))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{b^4\log(a+b\cos(x))}{(a^2-b^2)^2}+\frac{b^2}{4(a+b)(b-b\cos(x))}-\frac{b^2}{4(a-b)(b\cos(x)+b)}-\frac{b(a+2b)\log(b-b\cos(x))}{4(a+b)^2}+\frac{b(a-2b)\log(b\cos(x)+b)}{4(a-b)^2}}{b}$$

input
```
Int[Csc[x]^3/(a + b*Cos[x]),x]
```

output
```
-((b^2/(4*(a + b)*(b - b*Cos[x])) - b^2/(4*(a - b)*(b + b*Cos[x])) - (b*(a
 + 2*b)*Log[b - b*Cos[x]])/(4*(a + b)^2) + (b^4*Log[a + b*Cos[x]])/(a^2 -
b^2)^2 + ((a - 2*b)*b*Log[b + b*Cos[x]])/(4*(a - b)^2))/b)
```

3.31.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{-4b^3\ln\left(\frac{2\cos(x)b+2a}{\cos(x)+1}\right)-2(a-b)\left(-(a+2b)(a-b)\ln(\csc(x)-\cot(x))+\left(\cot(x)\csc(x)a-\frac{b\left(\cot^2(x)\right)}{2}-\frac{b\left(\csc^2(x)\right)}{2}\right)(a+b)\right)}{4(a-b)^2(a+b)^2}$
default	$\frac{1}{(4a+4b)(\cos(x)-1)}+\frac{(a+2b)\ln(\cos(x)-1)}{4(a+b)^2}-\frac{b^3\ln(a+\cos(x)b)}{(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(\cos(x)+1)}+\frac{(-a+2b)\ln(\cos(x)+1)}{4(a-b)^2}$
norman	$\frac{-\frac{1}{8(a+b)}+\frac{\tan^4\left(\frac{x}{2}\right)}{8a-8b}}{\tan\left(\frac{x}{2}\right)^2}-\frac{b^3\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{a^4-2a^2b^2+b^4}+\frac{(a+2b)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2+4ab+2b^2}$
risch	$\frac{ixa}{2a^2-4ab+2b^2}-\frac{ixb}{a^2-2ab+b^2}-\frac{ixa}{2(a^2+2ab+b^2)}-\frac{ixb}{a^2+2ab+b^2}+\frac{2ix\,b^3}{a^4-2a^2b^2+b^4}-\frac{a\,e^{3ix}-2\,e^{2ix}b+a\,e^{ix}}{(e^{2ix}-1)^2(-a^2+b^2)}-\frac{\ln(e^{ix}+1}{2(a^2-2ab-}$

input
```
int(csc(x)^3/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*b^3*ln((2*cos(x)*b+2*a)/(cos(x)+1))-2*(a-b)*(-(a+2*b)*(a-b)*ln(csc
(x)-cot(x))+(cot(x)*csc(x)*a-1/2*b*cot(x)^2-1/2*b*csc(x)^2)*(a+b)))/(a-b)^
2/(a+b)^2
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(87) = 174$.

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.97

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx$$
$$= \frac{2\,a^2b-2\,b^3-2\,(a^3-ab^2)\cos{(x)}+4\,\left(b^3\cos{(x)}^2-b^3\right)\log{(-b\cos{(x)}-a)}-\left(a^3-3\,ab^2-2\,b^3-(a^3-3\right.}{4\left(a^4-2\,a^2b^2+b^4-\right.}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x) + 4*(b^3*cos(x)^2 - b^3)*log
(-b*cos(x) - a) - (a^3 - 3*a*b^2 - 2*b^3 - (a^3 - 3*a*b^2 - 2*b^3)*cos(x)^
2)*log(1/2*cos(x) + 1/2) + (a^3 - 3*a*b^2 + 2*b^3 - (a^3 - 3*a*b^2 + 2*b^3
)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*
b^2 + b^4)*cos(x)^2)
```

3.31.6 Sympy [F]

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^3{(x)}}{a+b\cos{(x)}}\,dx$$

input
```
integrate(csc(x)**3/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)**3/(a + b*cos(x)), x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\frac{b^3\log{(b\cos{(x)}+a)}}{a^4-2\,a^2b^2+b^4}-\frac{(a-2\,b)\log{(\cos{(x)}+1)}}{4\,(a^2-2\,ab+b^2)} + \frac{(a+2\,b)\log{(\cos{(x)}-1)}}{4\,(a^2+2\,ab+b^2)}+\frac{a\cos{(x)}-b}{2\left((a^2-b^2)\cos{(x)}^2-a^2+b^2\right)}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
-b^3*log(b*cos(x) + a)/(a^4 - 2*a^2*b^2 + b^4) - 1/4*(a - 2*b)*log(cos(x)
+ 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(cos(x) - 1)/(a^2 + 2*a*b + b^
2) + 1/2*(a*cos(x) - b)/((a^2 - b^2)*cos(x)^2 - a^2 + b^2)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.48

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\frac{b^4\log\left(|b\cos(x)+a|\right)}{a^4b-2\,a^2b^3+b^5} - \frac{(a-2\,b)\log\left(\cos(x)+1\right)}{4\,(a^2-2\,ab+b^2)} + \frac{(a+2\,b)\log\left(-\cos(x)+1\right)}{4\,(a^2+2\,ab+b^2)} - \frac{a^2b-b^3-(a^3-ab^2)\cos(x)}{2\,(a+b)^2(a-b)^2(\cos(x)+1)(\cos(x)-1)}$$

input
```
integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-b^4*log(abs(b*cos(x) + a))/(a^4*b - 2*a^2*b^3 + b^5) - 1/4*(a - 2*b)*log(
cos(x) + 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(-cos(x) + 1)/(a^2 + 2*
a*b + b^2) - 1/2*(a^2*b - b^3 - (a^3 - a*b^2)*cos(x))/((a + b)^2*(a - b)^2
*(cos(x) + 1)*(cos(x) - 1))
```

3.31.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = \ln\left(\cos(x)-1\right)\left(\frac{b}{4\,(a+b)^2}+\frac{1}{4\,(a+b)}\right) + \frac{\frac{b}{2\,(a^2-b^2)}-\frac{a\,\cos(x)}{2\,(a^2-b^2)}}{\sin(x)^2} - \frac{b^3\,\ln\left(a+b\,\cos(x)\right)}{a^4-2\,a^2\,b^2+b^4} - \frac{\ln\left(\cos(x)+1\right)\,(a-2\,b)}{4\,(a-b)^2}$$

input
```
int(1/(sin(x)^3*(a + b*cos(x))),x)
```

output
```
log(cos(x) - 1)*(b/(4*(a + b)^2) + 1/(4*(a + b))) + (b/(2*(a^2 - b^2)) - (
a*cos(x))/(2*(a^2 - b^2)))/sin(x)^2 - (b^3*log(a + b*cos(x)))/(a^4 + b^4 -
 2*a^2*b^2) - (log(cos(x) + 1)*(a - 2*b))/(4*(a - b)^2)
```

3.32 $\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$

3.32.1 Optimal result

Integrand size = 13, antiderivative size = 110

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3\left(a^2-b^2\right)^2} + \frac{(b-a\cos(x))\csc^3(x)}{3\left(a^2-b^2\right)}$$

output
```
2*b^4*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)-1
/3*(3*b^3+a*(2*a^2-5*b^2)*cos(x))*csc(x)/(a^2-b^2)^2+1/3*(b-a*cos(x))*csc(
x)^3/(a^2-b^2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = -\frac{2b^4\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{5/2}} + \frac{\left((-6a^3+9ab^2)\cos(x)+6b^3\cos(2x)+\left(2a^2-5b^2\right)(2b+a\cos(3x))\right)\csc^3(x)}{12(a-b)^2(a+b)^2}$$

input
```
Integrate[Csc[x]^4/(a + b*Cos[x]),x]
```

output
```
(-2*b^4*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) +
 (((-6*a^3 + 9*a*b^2)*Cos[x] + 6*b^3*Cos[2*x] + (2*a^2 - 5*b^2)*(2*b + a*C
os[3*x]))*Csc[x]^3)/(12*(a - b)^2*(a + b)^2)
```

3.32.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3175, 25, 3042, 3345, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(x-\frac{\pi}{2}\right)^4\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}-\frac{\int -\frac{\left(2a^2+2b\cos(x)a-3b^2\right)\csc^2(x)}{a+b\cos(x)}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(2a^2+2b\cos(x)a-3b^2\right)\csc^2(x)}{a+b\cos(x)}dx}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a^2-2b\sin\left(x-\frac{\pi}{2}\right)a-3b^2}{\cos\left(x-\frac{\pi}{2}\right)^2\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)}dx}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{-\frac{\int -\frac{3b^4}{a+b\cos(x)}dx}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3b^4\int \frac{1}{a+b\cos(x)}dx}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{3b^4\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{3138}$$

$$\frac{\frac{6b^4\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{6b^4\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\csc(x)\left(a\left(2a^2-5b^2\right)\cos(x)+3b^3\right)}{a^2-b^2}}{3\left(a^2-b^2\right)}+\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}$$

input
```
Int[Csc[x]^4/(a + b*Cos[x]),x]
```

output
```
((b - a*Cos[x])*Csc[x]^3)/(3*(a^2 - b^2)) + ((6*b^4*ArcTan[(Sqrt[a - b]*Ta
n[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - ((3*b^3 + a*
(2*a^2 - 5*b^2)*Cos[x])*Csc[x])/(a^2 - b^2))/(3*(a^2 - b^2))
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.32.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{a\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}-\frac{b\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}+3a\tan\left(\frac{x}{2}\right)-5b\tan\left(\frac{x}{2}\right)}{8(a-b)^{2}}-\frac{1}{24(a+b)\tan\left(\frac{x}{2}\right)^{3}}-\frac{3a+5b}{8(a+b)^{2}\tan\left(\frac{x}{2}\right)}+\frac{2b^{4}\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{(a-b)(a+b)}}$
risch	$-\frac{2i\left(3b^{3}e^{5ix}-3a\,b^{2}e^{4ix}+4a^{2}b\,e^{3ix}-10b^{3}e^{3ix}-6a^{3}e^{2ix}+12a\,b^{2}e^{2ix}+3b^{3}e^{ix}+2a^{3}-5a\,b^{2}\right)}{3(a^{4}-2a^{2}b^{2}+b^{4})(e^{2ix}-1)^{3}}-\frac{b^{4}\ln\left(e^{ix}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)^{2}}+$

input
```
int(csc(x)^4/(a+cos(x)*b),x,method=_RETURNVERBOSE)
```

output
```
1/8/(a-b)^2*(1/3*a*tan(1/2*x)^3-1/3*b*tan(1/2*x)^3+3*a*tan(1/2*x)-5*b*tan(
1/2*x))-1/24/(a+b)/tan(1/2*x)^3-1/8*(3*a+5*b)/(a+b)^2/tan(1/2*x)+2/(a-b)^2
/(a+b)^2*b^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/
2))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.

Time = 0.28 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.17

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \left[\frac{2\,a^4b-10\,a^2b^3+8\,b^5+2\,(2\,a^5-7\,a^3b^2+5\,ab^4)\cos{(x)}^3+3\,(b^4\cos{(x)}^2-b^4)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(\cdots}{\cdots}\right.\cdots}{6\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6-(a^6\cdots}\right.$$

input
```
integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
[1/6*(2*a^4*b - 10*a^2*b^3 + 8*b^5 + 2*(2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x
)^3 + 3*(b^4*cos(x)^2 - b^4)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 -
 b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(
b^2*cos(x)^2 + 2*a*b*cos(x) + a^2))*sin(x) + 6*(a^2*b^3 - b^5)*cos(x)^2 -
6*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
- (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*sin(x)), 1/3*(a^4*b - 5*a^
2*b^3 + 4*b^5 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x)^3 - 3*(b^4*cos(x)^2 -
 b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))*sin
(x) + 3*(a^2*b^3 - b^5)*cos(x)^2 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
os(x)^2)*sin(x))]
```

3.32.6 Sympy [F]

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \int \frac{\csc^4{(x)}}{a+b\cos{(x)}}\,dx$$

input
```
integrate(csc(x)**4/(a+b*cos(x)),x)
```

output
```
Integral(csc(x)**4/(a + b*cos(x)), x)
```

3.32.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(97) = 194$.

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}} + \frac{a^2\tan\left(\frac{1}{2}x\right)^3-2ab\tan\left(\frac{1}{2}x\right)^3+b^2\tan\left(\frac{1}{2}x\right)^3+9a^2\tan\left(\frac{1}{2}x\right)-24ab\tan\left(\frac{1}{2}x\right)+15b^2\tan\left(\frac{1}{2}x\right)}{24(a^3-3a^2b+3ab^2-b^3)} - \frac{9a\tan\left(\frac{1}{2}x\right)^2+15b\tan\left(\frac{1}{2}x\right)^2+a+b}{24(a^2+2ab+b^2)\tan\left(\frac{1}{2}x\right)^3}$$

input
```
integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x))/sqrt(a^2 - b^2)))*b^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
 + 1/24*(a^2*tan(1/2*x)^3 - 2*a*b*tan(1/2*x)^3 + b^2*tan(1/2*x)^3 + 9*a^2*
tan(1/2*x) - 24*a*b*tan(1/2*x) + 15*b^2*tan(1/2*x))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - 1/24*(9*a*tan(1/2*x)^2 + 15*b*tan(1/2*x)^2 + a + b)/((a^2 + 2*
a*b + b^2)*tan(1/2*x)^3)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx = \tan\left(\frac{x}{2}\right)\left(\frac{4}{8a-8b}-\frac{8a+8b}{(8a-8b)^2}\right)+\frac{\tan\left(\frac{x}{2}\right)^3}{3\,(8a-8b)} - \frac{\frac{a^2-2ab+b^2}{3\,(a+b)}-\frac{\tan\left(\frac{x}{2}\right)^2\,(-3a^3+a^2b+7ab^2-5b^3)}{(a+b)^2}}{\tan\left(\frac{x}{2}\right)^3\,(8a^2-16ab+8b^2)} + \frac{2b^4\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)\,(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}\,(a-b)^{3/2}}\right)}{(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(sin(x)^4*(a + b*cos(x))),x)
```

output
```
tan(x/2)*(4/(8*a - 8*b) - (8*a + 8*b)/(8*a - 8*b)^2) + tan(x/2)^3/(3*(8*a
- 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) - (tan(x/2)^2*(7*a*b^2 + a^2*b
- 3*a^3 - 5*b^3))/(a + b)^2)/(tan(x/2)^3*(8*a^2 - 16*a*b + 8*b^2)) + (2*b^
4*atan((tan(x/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/
((a + b)^(5/2)*(a - b)^(5/2))
```

3.33 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx$

3.33.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{10ae^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d} + \frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

output
```
-2/7*a*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+2/9*b*(e*sin(d*x+c))^(9/2)/d/e-
10/21*a*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(d*x
+c))^(1/2)-10/21*a*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

3.33.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{e^3\Big(-120a\operatorname{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+(21b-138a\cos(c+dx)-28b\cos(2(c+dx))+}{252d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(e^3*(-120*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (21*b - 138*a*Cos[c + d
*x] - 28*b*Cos[2*(c + d*x)] + 18*a*Cos[3*(c + d*x)] + 7*b*Cos[4*(c + d*x)]
)*Sqrt[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(252*d*Sqrt[Sin[c + d*x]])
```

3.33.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3148, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{7/2}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3148}$$

$$a\int (e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3042}$$

$$a\int (e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \; 3042$$

$$a\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \; 3121$$

$$a\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \; 3042$$

$$a\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

$$\Big\downarrow \; 3120$$

$$a\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

input
```
Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(2*b*(e*Sin[c + d*x])^(9/2))/(9*d*e) + a*((-2*e*Cos[c + d*x]*(e*Sin[c + d*
x])^(5/2))/(7*d) + (5*e^2*((2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c + d*x]*Sqrt[e*Sin[c +
 d*x]])/(3*d)))/7)
```

3.33 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx$

3.33.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.33.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}}{9e}-\frac{e^4 a\left(-6\left(\sin^5(dx+c)\right)+5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\left(\sin^3(dx+c)\right)+10\sin(dx+c)\right.}{21\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\,e^4\left(-6(\sin^5(dx+c))+5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4(\sin^3(dx+c))+10\sin(dx+c)\right)}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/9/e*b*(e*sin(d*x+c))^(9/2)-1/21*e^4*a*(-6*sin(d*x+c)^5+5*(1-sin(d*x+c))
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1
/2),1/2*2^(1/2))-4*sin(d*x+c)^3+10*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(
1/2))/d
```

3.33.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{15\sqrt{2}a\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}a\sqrt{i\,e}e^3\text{weierst}}{}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/63*(15*sqrt(2)*a*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 15*sqrt(2)*a*sqrt(I*e)*e^3*weierstrassPInverse(4, 0, co
s(d*x + c) - I*sin(d*x + c)) + 2*(7*b*e^3*cos(d*x + c)^4 + 9*a*e^3*cos(d*x
 + c)^3 - 14*b*e^3*cos(d*x + c)^2 - 24*a*e^3*cos(d*x + c) + 7*b*e^3)*sqrt(
e*sin(d*x + c)))/d
```

3.33.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.33.7 Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2), x)
```

3.33.8 Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2), x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \int (e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)), x)
```

3.34 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx$

3.34.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx=\frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

output
```
-2/5*a*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+2/7*b*(e*sin(d*x+c))^(7/2)/d/e-
6/5*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+
c)^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx=\frac{2(e\sin(c+dx))^{5/2}\left(-21aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+\sin^{\frac{3}{2}}(c+dx)\left(-7a\cos(c+dx)+5b\sin^2\right.\right.}{35d\sin^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*(e*Sin[c + d*x])^(5/2)*(-21*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + Sin
[c + d*x]^(3/2)*(-7*a*Cos[c + d*x] + 5*b*Sin[c + d*x]^2)))/(35*d*Sin[c + d
*x]^(5/2))
```

3.34.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3148, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{5/2}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3148}$$

$$a\int (e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$a\int (e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3121}$$

$$a\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

input
```
Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*b*(e*Sin[c + d*x])^(7/2))/(7*d*e) + a*((6*e^2*EllipticE[(c - Pi/2 + d*x
)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*e*Cos[c + d*x]
*(e*Sin[c + d*x])^(3/2))/(5*d))
```

3.34.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.34.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.71

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}}{7e}-\frac{e^3a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\cdots\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\,e^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\cdots\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7/e*b*(e*sin(d*x+c))^(7/2)-1/5*e^3*a*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-
3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^4+2*sin(d*x+c)^2)/cos(d*x+c)
/(e*sin(d*x+c))^(1/2))/d
```

3.34.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.27

$$\int(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx=\frac{21i\sqrt{2}a\sqrt{-i\,e}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))\cdots}{\cdots}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/35*(21*I*sqrt(2)*a*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*a*sqrt(I*e)*e^2*w
eierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
c))) - 2*(5*b*e^2*cos(d*x + c)^2 + 7*a*e^2*cos(d*x + c) - 5*b*e^2)*sqrt(e*
sin(d*x + c))*sin(d*x + c))/d
```

3.34.6 Sympy [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (e\sin(c+dx))^{\frac{5}{2}}\,(a+b\cos(c+dx))\,\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x)), x)
```

3.34.7 Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

3.34.8 Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx = \int (e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)), x)
```

3.35 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx$

3.35.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx=\frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

output
```
2/5*b*(e*sin(d*x+c))^(5/2)/d/e-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*
sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2/3*a*e*cos(d*x+c)*(e*sin(d*x+c))^
(1/2)/d
```

3.35.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx=\frac{2(e\sin(c+dx))^{3/2}\left(-5a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)+\sqrt{\sin(c+dx)}(-5a\cos(c+dx)+\right.}{15d\sin^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*(e*Sin[c + d*x])^(3/2)*(-5*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + Sqrt
[Sin[c + d*x]]*(-5*a*Cos[c + d*x] + 3*b*Sin[c + d*x]^2)))/(15*d*Sin[c + d*
x]^(3/2))
```

3.35.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3148, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{3/2}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3148}$$

$$a\int (e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$a\int (e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3121}$$

$$a\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

input
```
Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*b*(e*Sin[c + d*x])^(5/2))/(5*d*e) + a*((2*e^2*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c + d*x]
*Sqrt[e*Sin[c + d*x]])/(3*d))
```

3.35.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.35.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{5}{2}}}{5e}-\frac{e^2a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\,e^2\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2(\sin^3(dx+c))+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b(e\sin(dx+c))}{5de}$

input
```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5/e*b*(e*sin(d*x+c))^(5/2)-1/3*e^2*a*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*
sin(d*x+c)^3+2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.35.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \frac{5\sqrt{2}a\sqrt{-i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}a\sqrt{i\,e}e\text{weierstrass}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/15*(5*sqrt(2)*a*sqrt(-I*e)*e*weierstrassPInverse(4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*sqrt(2)*a*sqrt(I*e)*e*weierstrassPInverse(4, 0, cos(d*x
+ c) - I*sin(d*x + c)) - 2*(3*b*e*cos(d*x + c)^2 + 5*a*e*cos(d*x + c) - 3*
b*e)*sqrt(e*sin(d*x + c)))/d
```

3.35.6 Sympy [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\cos{(c+dx)})\,\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x)), x)
```

3.35.7 Maxima [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{3}{2}}\,\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

3.35.8 Giac [F]

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (b\cos{(dx+c)}+a)(e\sin{(dx+c)})^{\frac{3}{2}}\,\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx = \int (e\,\sin\left(c+d\,x\right))^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)), x)
```

3.36 $\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx$

3.36.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

output
```
2/3*b*(e*sin(d*x+c))^(3/2)/d/e-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin
(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2\sqrt{e\sin(c+dx)}\left(-3aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b\sin^{\frac{3}{2}}(c+dx)\right)}{3d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*Sqrt[e*Sin[c + d*x]]*(-3*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + b*Sin[
c + d*x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

3.36.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\sin(c+dx)}(a+b\cos(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3148}$$

$$a\int \sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

input
```
Int[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c
+ d*x]]) + (2*b*(e*Sin[c + d*x])^(3/2))/(3*d*e)
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.36.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}}{3e}-\frac{ae\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{ae\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b(e\sin(dx+c))^{\frac{3}{2}}}{3de}$

input
```
int((a+cos(d*x+c)*b)*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/3*b/e*(e*sin(d*x+c))^(3/2)-a*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)*(2*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-Ellipt
icF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.36.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx$$
$$= \frac{3i\sqrt{2}a\sqrt{-i\,e}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))-3i\sqrt{2}a\sqrt{i\,e}\text{w}}{}$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*I*sqrt(2)*a*sqrt(-I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a*sqrt(I*e)*weierstrassZ
eta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sq
rt(e*sin(d*x + c))*b*sin(d*x + c))/d
```

3.36.6 Sympy [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\sin{(c+dx)}}(a+b\cos{(c+dx)})\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x)), x)
```

3.36.7 Maxima [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \int (b\cos{(dx+c)}+a)\sqrt{e\sin{(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

3.36.8 Giac [F]

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx=\int (b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 13.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx=\frac{2\,b\,\sin(c+d\,x)\,\sqrt{e\,\sin(c+d\,x)}}{3\,d}+\frac{2\,a\,\sqrt{e\,\sin(c+d\,x)}\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{\sin(c+d\,x)}}$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x)),x)
```

output
```
(2*b*sin(c + d*x)*(e*sin(c + d*x))^(1/2))/(3*d) + (2*a*(e*sin(c + d*x))^(1
/2)*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/(d*sin(c + d*x)^(1/2))
```

3.37 $\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

3.37.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

output
```
-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1
/2)+2*b*(e*sin(d*x+c))^(1/2)/d/e
```

3.37.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2\left(-a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)}{d\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*(-(a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]) + b*Sin[c
+ d*x]))/(d*Sqrt[e*Sin[c + d*x]])
```

3.37.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3121}$$

$$\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

input
```
Int[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c
+ d*x]]) + (2*b*Sqrt[e*Sin[c + d*x]])/(d*e)
```

3.37.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.37.4 Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

method	result
default	$-\frac{a\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\sin(dx+c)\cos(dx+c)b}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$-\frac{a\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b\sqrt{e\sin(dx+c)}}{de}$
risch	$-\frac{ib\left(\mathrm{e}^{2i(dx+c)}-1\right)\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{d\sqrt{-ie\left(\mathrm{e}^{2i(dx+c)}-1\right)\mathrm{e}^{-i(dx+c)}}}-\frac{ia\sqrt{\mathrm{e}^{i(dx+c)}+1}\,\sqrt{-2\,\mathrm{e}^{i(dx+c)}+2}\,\sqrt{-\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{\mathrm{e}^{i(dx+c)}+1},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-ie\left(\mathrm{e}^{2i(dx+c)}-1\right)\mathrm{e}}}{d\sqrt{-ie\,\mathrm{e}^{3i(dx+c)}+ie\,\mathrm{e}^{i(dx+c)}}\,\sqrt{-ie\left(\mathrm{e}^{2i(dx+c)}-1\right)\mathrm{e}^{-i(dx+c)}}}$

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(a*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin
(d*x+c)*cos(d*x+c)*b)/d
```

3.37.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}a\sqrt{-i\,e}\text{weierstrassPInverse}(4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}a\sqrt{i\,e}\text{weierstrassPInverse}(4,0,\cos}{de}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*a*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + sqrt(2)*a*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*sqrt(e*sin(d*x + c))*b)/(d*e)
```

3.37.6 Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{a+b\cos{(c+dx)}}{\sqrt{e\sin{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))/sqrt(e*sin(c + d*x)), x)
```

3.37.7 Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{b\cos{(dx+c)}+a}{\sqrt{e\sin{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

3.37.8 Giac [F]

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{b\cos(dx+c)+a}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = -\frac{2\sqrt{\sin(c+d\,x)}\left(a\,\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|2\right)-b\sqrt{\sin(c+d\,x)}\right)}{d\sqrt{e\,\sin(c+d\,x)}}$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)
```

output
```
-(2*sin(c + d*x)^(1/2)*(a*ellipticF(pi/4 - c/2 - (d*x)/2, 2) - b*sin(c + d
*x)^(1/2)))/(d*(e*sin(c + d*x))^(1/2))
```

3.38 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}} \, dx$

3.38.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}} \, dx = -\frac{2b}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

output
```
-2*b/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+2*a*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/
2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}} \, dx = -\frac{2\left(b+a\cos(c+dx)-aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x] - a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c
 + d*x]]))/(d*e*Sqrt[e*Sin[c + d*x]])
```

3.38.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3148, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{(e\sin(c+dx))^{3/2}}dx-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3116}$$

$$a\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3121}$$

$$a\left(-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$a\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*b)/(d*e*Sqrt[e*Sin[c + d*x]]) + a*((-2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c
 + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e
^2*Sqrt[Sin[c + d*x]]))
```

3.38.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.38.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.59

method	result
default	$\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a-a\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$\frac{a\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a-a*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*a*c
os(d*x+c)^2-2*cos(d*x+c)*b)/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d
```

3.38.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}a\sqrt{-i\,e}\sin(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+}{}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*a*sqrt(I*e)*sin(d*
x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*si
n(d*x + c))) - 2*(a*cos(d*x + c) + b)*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x
 + c))
```

3.38.6 Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{a+b\cos{\left(c+dx \right)}}{\left(e\sin{\left(c+dx \right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(3/2), x)
```

3.38.7 Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{b\cos\left(dx+c\right)+a}{\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

3.38.8 Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{b\cos\left(dx+c\right)+a}{\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{a+b\,\cos\left(c+d\,x\right)}{\left(e\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

3.39 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

3.39.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}$$

output
```
-2/3*b/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)-
2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+c
))^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(b+a\cos(c+dx)+a\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x] + a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*
x]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

3.39.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3148, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3148}$$

$$a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{(e\sin(c+dx))^{5/2}}dx-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$a\left(\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3e^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}\right)-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*b)/(3*d*e*(e*Sin[c + d*x])^(3/2)) + a*((-2*Cos[c + d*x])/(3*d*e*(e*Sin
[c + d*x])^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(3*d*e^2*Sqrt[e*Sin[c + d*x]]))
```

3.39.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.39.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.22

method	result
default	$\frac{-\frac{2b}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^{3}(dx+c)\right)+2\sin(dx+c)\right)}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2(\sin^{3}(dx+c))+2\sin(dx+c)\right)}{3e^{2}\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b}{3de(e\sin(dx+c))^{\frac{3}{2}}}$

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*b/e/(e*sin(d*x+c))^(3/2)-1/3*a/e^2*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2
*sin(d*x+c)^3+2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/
d
```

3.39.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}a\cos\left(dx+c\right)^{2}-\sqrt{2}a\right)\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(d\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*((sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a)*sqrt(-I*e)*weierstrassPInverse
(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*a*cos(d*x + c)^2 - sqrt(2
)*a)*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) +
2*(a*cos(d*x + c) + b)*sqrt(e*sin(d*x + c)))/(d*e^3*cos(d*x + c)^2 - d*e^3
)
```

3.39.6 Sympy [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{a+b\cos{(c+dx)}}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(5/2), x)
```

3.39.7 Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

3.39.8 Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{b\cos{(dx+c)}+a}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{a+b\ \cos\left(c+d\,x\right)}{\left(e\ \sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

3.40 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx$

3.40.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2b}{5de(e\sin(c+dx))^{5/2}}-\frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} -\frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}$$

output
```
-2/5*b/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(5/2)-
6/5*a*cos(d*x+c)/d/e^3/(e*sin(d*x+c))^(1/2)+6/5*a*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \frac{-4b-7a\cos(c+dx)+3a\cos(3(c+dx))+12aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{5}{2}}(c+}{10de(e\sin(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

output
```
(-4*b - 7*a*Cos[c + d*x] + 3*a*Cos[3*(c + d*x)] + 12*a*EllipticE[(-2*c + P
i - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(10*d*e*(e*Sin[c + d*x])^(5/2))
```

3.40.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 3148, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3148}$$

$$a\int \frac{1}{(e\sin(c+dx))^{7/2}}dx-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \frac{1}{(e\sin(c+dx))^{7/2}}dx-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3116}$$

$$a\left(\frac{3\int \frac{1}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{3\int \frac{1}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3116}$$

$$a\left(\frac{3\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{3\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3121}$$

$$a\left(\frac{3\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\left(-\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{3\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right)}{5e^2}-\frac{2\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}\right)-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

input
```
Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*b)/(5*d*e*(e*Sin[c + d*x])^(5/2)) + a*((-2*Cos[c + d*x])/(5*d*e*(e*Sin
[c + d*x])^(5/2)) + (3*((-2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*
EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c +
 d*x]])))/(5*e^2))
```

3.40.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

3.40.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

method	result
default	$\frac{-\frac{2b}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right.\right.}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$\frac{a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)F\left(\sqrt{\right.\right.}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/5*b/e/(e*sin(d*x+c))^(5/2)+1/5*a/e^3*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
-3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^5-4*sin(d*x+c)^3-2*sin(d*x+
c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.40.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx =$$
$$-\frac{3\left(i\sqrt{2}a\cos\left(dx+c\right)^2-i\sqrt{2}a\right)\sqrt{-ie}\sin\left(dx+c\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(I*sqrt(2)*a*cos(d*x + c)^2 - I*sqrt(2)*a)*sqrt(-I*e)*sin(d*x + c)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x
+ c))) + 3*(-I*sqrt(2)*a*cos(d*x + c)^2 + I*sqrt(2)*a)*sqrt(I*e)*sin(d*x +
 c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d
*x + c))) + 2*(3*a*cos(d*x + c)^3 - 4*a*cos(d*x + c) - b)*sqrt(e*sin(d*x +
 c)))/((d*e^4*cos(d*x + c)^2 - d*e^4)*sin(d*x + c))
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.40.7 Maxima [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{b\cos\left(dx+c\right)+a}{\left(e\sin\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(7/2), x)
```

3.40.8 Giac [F]

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{b\cos(dx+c)+a}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)/(e*sin(d*x + c))^(7/2), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{a+b\,\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{7/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2), x)
```

3.41 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx$

3.41.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$\begin{aligned}\int (a+b\cos(c+dx))^2(e\sin(c&+dx))^{7/2}\,dx = \frac{10(11a^2+2b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}}\\ &-\frac{10(11a^2+2b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}\\ &-\frac{2(11a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}\\ &+\frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de}\end{aligned}$$

output
```
-2/77*(11*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+26/99*a*b*(e*sin(
d*x+c))^(9/2)/d/e+2/11*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(9/2)/d/e-10/231*
(11*a^2+2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*
sin(d*x+c))^(1/2)-10/231*(11*a^2+2*b^2)*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2
)/d
```

3.41.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{\left(\frac{1}{6}(924ab-6(506a^2+71b^2)\cos(c+dx)-1232ab\cos(2(c+dx))+396a^2\cos(3(c+dx))-1\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

output
```
((((924*a*b - 6*(506*a^2 + 71*b^2)*Cos[c + d*x] - 1232*a*b*Cos[2*(c + d*x)
] + 396*a^2*Cos[3*(c + d*x)] - 117*b^2*Cos[3*(c + d*x)] + 308*a*b*Cos[4*(c
 + d*x)] + 63*b^2*Cos[5*(c + d*x)])*Csc[c + d*x]^3)/6 - (40*(11*a^2 + 2*b^
2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2))*(e*Sin[c + d*x
])^(7/2))/(924*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{11}\int \frac{1}{2}\left(11a^2+13b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \left(11a^2+13b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{11}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(11a^2-13b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \textbf{3148}$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\int (e\sin(c+dx))^{7/2}dx+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\int (e\sin(c+dx))^{7/2}dx+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\int (e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{26ab(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+d}{7d}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$$\Big\downarrow \textbf{3042}$$

3.41. $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx$

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin(c+d}{7d}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$\Big\downarrow$ 3121

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin}{7d}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\sin}{7d}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

$\Big\downarrow$ 3120

$$\frac{1}{11}\left(\left(11a^2+2b^2\right)\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\,\text{co}}{}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(11*d*e) + ((26*a*b*(e*S
in[c + d*x])^(9/2))/(9*d*e) + (11*a^2 + 2*b^2)*((-2*e*Cos[c + d*x]*(e*Sin[
c + d*x])^(5/2))/(7*d) + (5*e^2*((2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c + d*x]*Sqrt[e*S
in[c + d*x]])/(3*d)))/7))/11
```

3.41. $\int(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx$

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

3.41.4 Maple [A] (verified)

Time = 9.72 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.31

method	result
default	$\frac{\frac{4ab(e\sin(dx+c))^{\frac{9}{2}}}{9e}-\frac{e^4\left(-42b^2\left(\cos^6(dx+c)\right)\sin(dx+c)-66a^2\left(\cos^4(dx+c)\right)\sin(dx+c)+72b^2\left(\cos^4(dx+c)\right)\sin(dx+c)+55\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+}\right.}{}}{}$
parts	$-\frac{a^2e^4\left(-6(\sin^5(dx+c))+5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4(\sin^3(dx+c))+10\sin(dx+c)\right.}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(4/9/e*a*b*(e*sin(d*x+c))^(9/2)-1/231*e^4*(-42*b^2*cos(d*x+c)^6*sin(d*x+c)
-66*a^2*cos(d*x+c)^4*sin(d*x+c)+72*b^2*cos(d*x+c)^4*sin(d*x+c)+55*(1-sin(d
*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+
c))^(1/2),1/2*2^(1/2))*a^2+10*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+176*a^2*c
os(d*x+c)^2*sin(d*x+c)-10*b^2*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d
*x+c))^(1/2))/d
```

3.41.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{15\sqrt{2}(11\,a^2+2\,b^2)\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}(11}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/693*(15*sqrt(2)*(11*a^2 + 2*b^2)*sqrt(-I*e)*e^3*weierstrassPInverse(4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(11*a^2 + 2*b^2)*sqrt(I*e)*e
^3*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(63*b^2*e^
3*cos(d*x + c)^5 + 154*a*b*e^3*cos(d*x + c)^4 - 308*a*b*e^3*cos(d*x + c)^2
 + 9*(11*a^2 - 12*b^2)*e^3*cos(d*x + c)^3 + 154*a*b*e^3 - 3*(88*a^2 - 5*b^
2)*e^3*cos(d*x + c))*sqrt(e*sin(d*x + c)))/d
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.41.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2), x)
```

3.41.8 Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx=\int (e\,\sin\left(c+d\,x\right))^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

3.42 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

3.42.1 Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{2(9a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}} - \frac{2(9a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d} + \frac{22ab(e\sin(c+dx))^{7/2}}{63de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de}$$

output
```
-2/45*(9*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+22/63*a*b*(e*sin(d
*x+c))^(7/2)/d/e+2/9*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2)/d/e-2/15*(9*a
^2+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x
)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(
d*x+c)^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx =$$
$$-\frac{(e\sin(c+dx))^{5/2}\left(84(9a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+(21(12a^2+b^2)\cos(c+dx)+5b(-36a+36a\right.}{630d\sin^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/630*((e*Sin[c + d*x])^(5/2)*(84*(9*a^2 + 2*b^2)*EllipticE[(-2*c + Pi -
2*d*x)/4, 2] + (21*(12*a^2 + b^2)*Cos[c + d*x] + 5*b*(-36*a + 36*a*Cos[2*(
c + d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5
/2))
```

3.42.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{9}\int \frac{1}{2}\left(9a^2+11b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \left(9a^2+11b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(9a^2-11b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\downarrow$ 3148

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\int (e\sin(c+dx))^{5/2}dx+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\int (e\sin(c+dx))^{5/2}dx+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\downarrow$ 3115

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\left(\frac{3}{5}e^2\int \sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\downarrow$ 3121

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx))}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+dx))}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

3.42. $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{9}\left(\left(9a^2+2b^2\right)\left(\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{22ab(e\sin(c+d}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(9*d*e) + ((22*a*b*(e*Si
n[c + d*x])^(7/2))/(7*d*e) + (9*a^2 + 2*b^2)*((6*e^2*EllipticE[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*e*Cos[c +
d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)))/9
```

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

3.42.4 Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.16

method	result
default	$\frac{\frac{4ab(e\sin(dx+c))^{\frac{7}{2}}}{7e}-\frac{e^3\left(10\left(\sin^6(dx+c)\right)b^2+54\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+12\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(}\right.}{}}{}$
parts	$-\frac{a^2e^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c\right.\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(4/7/e*a*b*(e*sin(d*x+c))^(7/2)-1/45*e^3*(10*sin(d*x+c)^6*b^2+54*(1-sin(d*
x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c
))^(1/2),1/2*2^(1/2))*a^2+12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-27*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x
+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-18*a^2*si
n(d*x+c)^4-14*sin(d*x+c)^4*b^2+18*a^2*sin(d*x+c)^2+4*b^2*sin(d*x+c)^2)/cos
(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.42.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \frac{21i\sqrt{2}(9a^2+2b^2)\sqrt{-ie}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/315*(21*I*sqrt(2)*(9*a^2 + 2*b^2)*sqrt(-I*e)*e^2*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(9
*a^2 + 2*b^2)*sqrt(I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*b^2*e^2*cos(d*x + c)^3 + 90*a*b*
e^2*cos(d*x + c)^2 - 90*a*b*e^2 + 21*(3*a^2 - b^2)*e^2*cos(d*x + c))*sqrt(
e*sin(d*x + c))*sin(d*x + c))/d
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.42.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

3.42.8 Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx=\int (b\cos{(dx+c)}+a)^2(e\sin{(dx+c)})^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx=\int (e\,\sin{(c+d\,x)})^{5/2}\,(a+b\,\cos{(c+d\,x)})^2\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

3.43 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$

3.43.1 Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{2(7a^2+2b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{18ab(e\sin(c+dx))^{5/2}}{35de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de}$$

output
```
18/35*a*b*(e*sin(d*x+c))^(5/2)/d/e+2/7*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(
5/2)/d/e-2/21*(7*a^2+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2/21*(7*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+
c))^(1/2)/d
```

3.43.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.76

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{\left(-\frac{1}{2}(5(28a^2+5b^2)\cos(c+dx)+3b(-28a+28a\cos(2(c+dx))+5b\cos(3(c+dx))))\csc(c+\right.}{105d}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

output
```
((-1/2*((5*(28*a^2 + 5*b^2)*Cos[c + d*x] + 3*b*(-28*a + 28*a*Cos[2*(c + d*
x)] + 5*b*Cos[3*(c + d*x)]))*Csc[c + d*x]) - (10*(7*a^2 + 2*b^2)*EllipticF
[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(1
05*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{7}\int \frac{1}{2}\left(7a^2+9b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \left(7a^2+9b\cos(c+dx)a+2b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(7a^2-9b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3148}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int (e\sin(c+dx))^{3/2}dx+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int (e\sin(c+dx))^{3/2}dx+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{1}{3}e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{e^2\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3042}$$

3.43. $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$

$$\frac{1}{7}\left((7a^2+2b^2)\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

$\big\downarrow$ 3120

$$\frac{1}{7}\left((7a^2+2b^2)\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{18ab(e\sin(c\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(7*d*e) + ((18*a*b*(e*Si
n[c + d*x])^(5/2))/(5*d*e) + (7*a^2 + 2*b^2)*((2*e^2*EllipticF[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)))/7
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

3.43.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.49

method	result
default	$-\frac{e^2\left(30b^2\left(\cos^4(dx+c)\right)\sin(dx+c)+35\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+10\sqrt{1-\sin(dx+c}\right.}{}$
parts	$-\frac{a^2e^2\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^2e^2\left(3\left(\sin^5(\right.\right.}{}$

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*(30*b^2*cos(d*x+c)^4*sin(d*x+c)
+35*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+10*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b
^2+84*a*b*cos(d*x+c)^3*sin(d*x+c)+70*a^2*cos(d*x+c)^2*sin(d*x+c)-10*b^2*co
s(d*x+c)^2*sin(d*x+c)-84*a*b*cos(d*x+c)*sin(d*x+c))/d
```

3.43.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{5\sqrt{2}(7a^2+2b^2)\sqrt{-i\,e}e\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(7a^2+$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/105*(5*sqrt(2)*(7*a^2 + 2*b^2)*sqrt(-I*e)*e*weierstrassPInverse(4, 0, co
s(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(7*a^2 + 2*b^2)*sqrt(I*e)*e*weier
strassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(15*b^2*e*cos(d*x
+ c)^3 + 42*a*b*e*cos(d*x + c)^2 - 42*a*b*e + 5*(7*a^2 - b^2)*e*cos(d*x +
c))*sqrt(e*sin(d*x + c)))/d
```

3.43.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (e\sin(c+dx))^{\frac{3}{2}}\,(a+b\cos(c+dx))^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**2, x)
```

3.43.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

3.43.8 Giac [F]

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

3.44 $\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$

3.44.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} + \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de}$$

output
```
14/15*a*b*(e*sin(d*x+c))^(3/2)/d/e+2/5*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(
3/2)/d/e-2/5*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))
^(1/2)/d/sin(d*x+c)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \frac{2\sqrt{e\sin(c+dx)}\left(-3(5a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b(10a+3b\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)\right)}{15d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*Sqrt[e*Sin[c + d*x]]*(-3*(5*a^2 + 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/
4, 2] + b*(10*a + 3*b*Cos[c + d*x])*Sin[c + d*x]^(3/2)))/(15*d*Sqrt[Sin[c
+ d*x]])
```

3.44.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{5}\int \frac{1}{2}\left(5a^2+7b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \left(5a^2+7b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(5a^2-7b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\int\sqrt{e\sin(c+dx)}dx+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3121}$$

$$\frac{1}{5}\left(\frac{\left(5a^2+2b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{\left(5a^2+2b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{2\left(5a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

input
```
Int[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(5*d*e) + ((2*(5*a^2 + 2
*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c
 + d*x]]) + (14*a*b*(e*Sin[c + d*x])^(3/2))/(3*d*e))/5
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(130) = 260$.

Time = 3.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.39

method	result
parts	$-\frac{a^2 e\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d} - \frac{2b^2e\left(2\sqrt{1-\sin(dx\right.}{}$
default	$-\frac{e\left(30\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+12\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c\right.\right.}{}$

input
```
int((a+cos(d*x+c)*b)^2*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(2*Ell
ipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/
2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2/5*b^2*e*(2*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/
2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)^4-cos(d*x+c)^2)/
cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d+4/3*a*b*(e*sin(d*x+c))^(3/2)/d/e
```

3.44.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx =$$
$$-\frac{3\sqrt{2}(-5i\,a^2-2i\,b^2)\sqrt{-i\,e}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))}{}$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(3*sqrt(2)*(-5*I*a^2 - 2*I*b^2)*sqrt(-I*e)*weierstrassZeta(4, 0, wei
erstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(5*I*a^
2 + 2*I*b^2)*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos
(d*x + c) - I*sin(d*x + c))) - 2*(3*b^2*cos(d*x + c) + 10*a*b)*sqrt(e*sin(
d*x + c))*sin(d*x + c))/d
```

3.44.6 Sympy [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\sin{(c+dx)}}(a+b\cos{(c+dx)})^2\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2, x)
```

3.44.7 Maxima [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int (b\cos(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

3.44.8 Giac [F]

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int (b\cos(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int \sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))^2\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

3.45 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

3.45.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} + \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de}$$

output
```
-2/3*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*
sin(d*x+c))^(1/2)+10/3*a*b*(e*sin(d*x+c))^(1/2)/d/e+2/3*b*(a+b*cos(d*x+c))
*(e*sin(d*x+c))^(1/2)/d/e
```

3.45.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{-2(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+2b(6a+b\cos(c+dx))\sin(c+dx)}{3d\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(-2*(3*a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]
 + 2*b*(6*a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[e*Sin[c + d*x]])
```

3.45.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3171, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{3}\int \frac{3a^2+5b\cos(c+dx)a+2b^2}{2\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+5b\cos(c+dx)a+2b^2}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^2-5b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$\big\downarrow$ 3121

$$\frac{1}{3}\left(\frac{\left(3a^2+2b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{\left(3a^2+2b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

$\big\downarrow$ 3120

$$\frac{1}{3}\left(\frac{2\left(3a^2+2b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{3de}$$

input
```
Int[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*d*e) + ((2*(3*a^2 + 2*b
^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c +
 d*x]]) + (10*a*b*Sqrt[e*Sin[c + d*x]])/(d*e))/3
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

3.45.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

method	result
default	$-\frac{3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F}{3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
parts	$-\frac{a^2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{b^2\left(-\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F}{3}\right.}{\cos(dx+c)\sqrt{e}}$

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2
+2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c)-12*a*b
*cos(d*x+c)*sin(d*x+c))/d
```

3.45.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}(3a^2+2b^2)\sqrt{-i\,e}\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(3a^2+2b^2)\sqrt{i\,e}\,\text{weierst}}{3\,de}$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(3*a^2 + 2*b^2)*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + sqrt(2)*(3*a^2 + 2*b^2)*sqrt(I*e)*weierstrassPInv
erse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(b^2*cos(d*x + c) + 6*a*b)*s
qrt(e*sin(d*x + c)))/(d*e)
```

3.45.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/sqrt(e*sin(c + d*x)), x)
```

3.45.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

3.45.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\cos(c+d\,x))^2}{\sqrt{e\sin(c+d\,x)}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

3.46 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

3.46.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}}{de^3}$$

output
```
-2*a*b*(e*sin(d*x+c))^(3/2)/d/e^3-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/
(e*sin(d*x+c))^(1/2)+2*(a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin
(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \frac{-4ab-2(a^2+b^2)\cos(c+dx)+2(a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\sqrt{\sin(c}{de\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-4*a*b - 2*(a^2 + b^2)*Cos[c + d*x] + 2*(a^2 + 2*b^2)*EllipticE[(-2*c + P
i - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]])/(d*e*Sqrt[e*Sin[c + d*x]])
```

3.46.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3170, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{2\int \frac{1}{2}\left(a^2+3b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \left(a^2+3b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a^2-3b\sin\left(c+dx-\frac{\pi}{2}\right)a+2b^2\right)dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3148}$$

$$-\frac{\left(a^2+2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2+2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{\frac{\left(a^2+2b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{(a^2+2b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{2(a^2+2b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(d*e*Sqrt[e*Sin[c + d*x]])
- ((2*(a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/(d*Sqrt[Sin[c + d*x]]) + (2*a*b*(e*Sin[c + d*x])^(3/2))/(d*e))/e^2
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(140) = 280$.

Time = 3.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.40

method	result
default	$\frac{2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2+4\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{\right.}{}}{}$
parts	$\frac{a^2\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{\right.}\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+
4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-2*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-
sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*a^2*cos(d*x+c)^2-2*b^2*cos(d*x+c)^2-4
*cos(d*x+c)*a*b)/d
```

3.46.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,a^2-2i\,b^2)\sqrt{-i\,e}\sin(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*(-I*a^2 - 2*I*b^2)*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*a^2
 + 2*I*b^2)*sqrt(I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*a*b + (a^2 + b^2)*cos(d*x
+ c))*sqrt(e*sin(d*x + c)))/(d*e^2*sin(d*x + c))
```

3.46.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(3/2), x)
```

3.46.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

3.46.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos(c+d\,x))^2}{(e\sin(c+d\,x))^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

3.47 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

3.47.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} + \frac{2(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3}$$

output
```
-2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(3/2)-2/3*(a^2-2
*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+c
))^(1/2)-2/3*a*b*(e*sin(d*x+c))^(1/2)/d/e^3
```

3.47.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2\left(2ab+(a^2+b^2)\cos(c+dx)+(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(2*a*b + (a^2 + b^2)*Cos[c + d*x] + (a^2 - 2*b^2)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

3.47.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3170, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{2\int -\frac{a^2-b\cos(c+dx)a-2b^2}{2\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2-b\cos(c+dx)a-2b^2}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2+b\sin\left(c+dx-\frac{\pi}{2}\right)a-2b^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3148}$$

$$\frac{(a^2-2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-2b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{(a^2-2b^2)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2-2b^2)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2(a^2-2b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(3*d*e*(e*Sin[c + d*x])^(3/
2)) + ((2*(a^2 - 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(d*Sqrt[e*Sin[c + d*x]]) - (2*a*b*Sqrt[e*Sin[c + d*x]])/(d*e))/(3*e^2)
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

3.47.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.63

method	result
default	$\frac{-\frac{4ab}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-2b^2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right.}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$
parts	$-\frac{a^2\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}+\frac{2b^2\left(\sqrt{1-\sin(dx-\right.}{}$

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-4/3*a*b/e/(e*sin(d*x+c))^(3/2)-1/3/e^2*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a
^2-2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*Elli
pticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+2*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^
2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.47.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}(a^2-2\,b^2)\cos\left(dx+c\right)^2-\sqrt{2}(a^2-2\,b^2)\right)\sqrt{-i\,e}\text{weierstrassPInverse}(4,0,c}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*((sqrt(2)*(a^2 - 2*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^2 - 2*b^2))*sqrt(-
I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(
a^2 - 2*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^2 - 2*b^2))*sqrt(I*e)*weierstrass
PInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(2*a*b + (a^2 + b^2)*cos
(d*x + c))*sqrt(e*sin(d*x + c)))/(d*e^3*cos(d*x + c)^2 - d*e^3)
```

3.47.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\left(a+b\cos\left(c+dx\right)\right)^2}{\left(e\sin\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(5/2), x)
```

3.47.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\left(b\cos\left(dx+c\right)+a\right)^2}{\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

3.47.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^2}{(e\,\sin(c+d\,x))^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)
```

3.48 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx$

3.48.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}$$

output

```
-2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*b/d/
e^3/(e*sin(d*x+c))^(1/2)-2/5*(3*a^2-2*b^2)*cos(d*x+c)/d/e^3/(e*sin(d*x+c))
^(1/2)+2/5*(3*a^2-2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(
1/2)/d/e^4/sin(d*x+c)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{8ab+(7a^2+2b^2)\cos(c+dx)-3a^2\cos(3(c+dx))+2b^2\cos(3(c+dx))-4(3a^2-2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2\right.}{10de(e\sin(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

output
```
-1/10*(8*a*b + (7*a^2 + 2*b^2)*Cos[c + d*x] - 3*a^2*Cos[3*(c + d*x)] + 2*b
^2*Cos[3*(c + d*x)] - 4*(3*a^2 - 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2
]*Sin[c + d*x]^(5/2))/(d*e*(e*Sin[c + d*x])^(5/2))
```

3.48.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3170, 27, 3042, 3148, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{2\int -\frac{3a^2+b\cos(c+dx)a-2b^2}{2(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a^2+b\cos(c+dx)a-2b^2}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a^2-b\sin\left(c+dx-\frac{\pi}{2}\right)a-2b^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3148}$$

$$\frac{(3a^2-2b^2)\int \frac{1}{(e\sin(c+dx))^{3/2}}dx-\frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3a^2-2b^2)\int \frac{1}{(e\sin(c+dx))^{3/2}}dx - \frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{(3a^2-2b^2)\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2} - \frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right) - \frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3a^2-2b^2)\left(-\frac{\int \sqrt{e\sin(c+dx)}dx}{e^2} - \frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right) - \frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{(3a^2-2b^2)\left(-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}} - \frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right) - \frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3a^2-2b^2)\left(-\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{e^2\sqrt{\sin(c+dx)}} - \frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right) - \frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{(3a^2-2b^2)\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}\right) - \frac{2ab}{de\sqrt{e\sin(c+dx)}}}{5e^2} - \frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(5*d*e*(e*Sin[c + d*x])^(5/
2)) + ((-2*a*b)/(d*e*Sqrt[e*Sin[c + d*x]]) + (3*a^2 - 2*b^2)*((-2*Cos[c +
d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqr
t[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]])))/(5*e^2)
```

3.48. $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx$

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

3.48. $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx$

3.48.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.13

method	result
default	$-\frac{4ab}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^{2}-4\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right.}{}$
parts	$\frac{a^{2}\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)F\left(\right.\right.}{5e^{3}\sin(dx+c)^{3}\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-4/5*a*b/e/(e*sin(d*x+c))^(5/2)+1/5/e^3*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
*a^2-4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*Ellipt
icE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-3*(1-sin(d*x+c))^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
*a^2+2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*Ellipt
icF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+6*a^2*cos(d*x+c)^4*sin(d*x+c)-4*
b^2*cos(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^2*cos(d*x+c)
^2*sin(d*x+c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.48.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.41

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \frac{\left(\sqrt{2}(-3i\,a^2+2i\,b^2)\cos\left(dx+c\right)^2+\sqrt{2}(3i\,a^2-2i\,b^2)\right)\sqrt{-i\,e}\sin\left(dx+c\right)\text{weiers}}{}$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/5*((sqrt(2)*(-3*I*a^2 + 2*I*b^2)*cos(d*x + c)^2 + sqrt(2)*(3*I*a^2 - 2*I
*b^2))*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(3*I*a^2 - 2*I*b^2)*cos(d*
x + c)^2 + sqrt(2)*(-3*I*a^2 + 2*I*b^2))*sqrt(I*e)*sin(d*x + c)*weierstras
sZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*
((3*a^2 - 2*b^2)*cos(d*x + c)^3 - 2*a*b - (4*a^2 - b^2)*cos(d*x + c))*sqrt
(e*sin(d*x + c)))/((d*e^4*cos(d*x + c)^2 - d*e^4)*sin(d*x + c))
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.48.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(7/2), x)
```

3.48.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^2/(e*sin(d*x + c))^(7/2), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2), x)
```

3.49 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx$

3.49.1 Optimal result

Integrand size = 25, antiderivative size = 242

$$\begin{aligned}\int (a+b\cos(c+dx))^3(e\sin(c&+dx))^{7/2}\,dx = \frac{10a(11a^2+6b^2)\,e^4\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}}\\ &- \frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}\\ &- \frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}\\ &+ \frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de}+\frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de}\\ &+ \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de}\end{aligned}$$

output
```
-2/77*a*(11*a^2+6*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+2/1287*b*(177*a
^2+44*b^2)*(e*sin(d*x+c))^(9/2)/d/e+34/143*a*b*(a+b*cos(d*x+c))*(e*sin(d*x
+c))^(9/2)/d/e+2/13*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(9/2)/d/e-10/231*a
*(11*a^2+6*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e
*sin(d*x+c))^(1/2)-10/231*a*(11*a^2+6*b^2)*e^3*cos(d*x+c)*(e*sin(d*x+c))^(
1/2)/d
```

3.49.2 Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.85

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \frac{\left(154b(78a^2+11b^2)\csc^3(c+dx)+\frac{1}{3}(-156a(506a^2+213b^2)\cos(c+dx)-77b(624a^2+73b^2)\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]
```

output
```
((154*b*(78*a^2 + 11*b^2)*Csc[c + d*x]^3 + ((-156*a*(506*a^2 + 213*b^2)*Co
s[c + d*x] - 77*b*(624*a^2 + 73*b^2)*Cos[2*(c + d*x)] + 234*a*(44*a^2 - 39
*b^2)*Cos[3*(c + d*x)] - 154*b*(-78*a^2 + b^2)*Cos[4*(c + d*x)] + 4914*a*b
^2*Cos[5*(c + d*x)] + 693*b^3*Cos[6*(c + d*x)])*Csc[c + d*x]^3)/3 - (2080*
a*(11*a^2 + 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2)
)*(e*Sin[c + d*x])^(7/2))/(48048*d)
```

3.49.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 3171}$$

$$\frac{2}{13}\int \frac{1}{2}(a+b\cos(c+dx))\left(13a^2+17b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$$\downarrow 27$$

$$\frac{1}{13}\int (a+b\cos(c+dx))\left(13a^2+17b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{7/2}dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$$\downarrow 3042$$

$$\frac{1}{13}\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(13a^2+17b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$$\downarrow 3341$$

$$\frac{1}{13}\left(\frac{2}{11}\int \frac{1}{2}\left(13a\left(11a^2+6b^2\right)+b\left(177a^2+44b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{7/2}dx+\frac{34ab(e\sin(c+dx))^{9/2}(a+b}{11de}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$$\downarrow 27$$

$$\frac{1}{13}\left(\frac{1}{11}\int \left(13a\left(11a^2+6b^2\right)+b\left(177a^2+44b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{7/2}dx+\frac{34ab(e\sin(c+dx))^{9/2}(a+b\cos}{11de}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$$\downarrow 3042$$

$$\frac{1}{13}\left(\frac{1}{11}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(13a\left(11a^2+6b^2\right)-b\left(177a^2+44b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{34ab(e\sin(c+}{}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$$\downarrow 3148$$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a\left(11a^2+6b^2\right)\int (e\sin(c+dx))^{7/2}dx+\frac{2b\left(177a^2+44b^2\right)(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{34ab(e\sin(c+dx))^{9/2}}{11d}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$$\downarrow 3042$$

3.49. $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\int(e\sin(c+dx))^{7/2}dx+\frac{2b(177a^2+44b^2)(e\sin(c+dx))^{9/2}}{9de}\right)+\frac{34ab(e\sin(c+dx))^{9/2}}{11d\ldots}\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$
$$\Big\downarrow \text{3115}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\int(e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(177a^2+44b^2)(\ldots}{9de\ldots}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\int(e\sin(c+dx))^{3/2}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}\right)+\frac{2b(177a^2+44b^2)(\ldots}{9de\ldots}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$
$$\Big\downarrow \text{3115}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\,s\ldots}{7d\ldots}\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+dx)(e\,s\ldots}{7d\ldots}\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$
$$\Big\downarrow \text{3121}$$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+\ldots}{\ldots}\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$
$$\Big\downarrow \text{3042}$$

3.49. $\int(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx$

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)-\frac{2e\cos(c+\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

$\big\downarrow$ 3120

$$\frac{1}{13}\left(\frac{1}{11}\left(13a(11a^2+6b^2)\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right.\right.\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(9/2))/(13*d*e) + ((34*a*b*(a
 + b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(11*d*e) + ((2*b*(177*a^2 + 44*
b^2)*(e*Sin[c + d*x])^(9/2))/(9*d*e) + 13*a*(11*a^2 + 6*b^2)*((-2*e*Cos[c
+ d*x]*(e*Sin[c + d*x])^(5/2))/(7*d) + (5*e^2*((2*e^2*EllipticF[(c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Cos[c +
 d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)))/7))/11)/13
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

3.49.4 Maple [A] (verified)

Time = 39.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}\left(9b^2\left(\cos^2(dx+c)\right)+39a^2+4b^2\right)}{117e}-\frac{e^4a\left(-126b^2\left(\cos^6(dx+c)\right)\sin(dx+c)-66a^2\left(\cos^4(dx+c)\right)\sin(dx+c)+216b^2\left(\cos^4(dx+c)\right)\sin(d\right.}{}}{}$
parts	$-\frac{a^3e^4\left(-6\left(\sin^5(dx+c)\right)+5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-4\left(\sin^3(dx+c)\right)+10\sin(dx+c)\right.}{21\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/117/e*b*(e*sin(d*x+c))^(9/2)*(9*b^2*cos(d*x+c)^2+39*a^2+4*b^2)-1/231*e^
4*a*(-126*b^2*cos(d*x+c)^6*sin(d*x+c)-66*a^2*cos(d*x+c)^4*sin(d*x+c)+216*b
^2*cos(d*x+c)^4*sin(d*x+c)+55*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+30*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))*b^2+176*a^2*cos(d*x+c)^2*sin(d*x+c)-30*b^2*cos(d*
x+c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.49.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \frac{195\sqrt{2}(11a^3+6ab^2)\sqrt{-i\,e}e^3\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+195\sqrt{}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/9009*(195*sqrt(2)*(11*a^3 + 6*a*b^2)*sqrt(-I*e)*e^3*weierstrassPInverse(
4, 0, cos(d*x + c) + I*sin(d*x + c)) + 195*sqrt(2)*(11*a^3 + 6*a*b^2)*sqrt
(I*e)*e^3*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(69
3*b^3*e^3*cos(d*x + c)^6 + 2457*a*b^2*e^3*cos(d*x + c)^5 + 77*(39*a^2*b -
14*b^3)*e^3*cos(d*x + c)^4 + 117*(11*a^3 - 36*a*b^2)*e^3*cos(d*x + c)^3 -
77*(78*a^2*b - b^3)*e^3*cos(d*x + c)^2 - 39*(88*a^3 - 15*a*b^2)*e^3*cos(d*
x + c) + 77*(39*a^2*b + 4*b^3)*e^3)*sqrt(e*sin(d*x + c)))/d
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.49.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2), x)
```

3.49.8 Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx = \int (e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

3.50 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx$

3.50.1 Optimal result

Integrand size = 25, antiderivative size = 202

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \frac{2a(3a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d} + \frac{2b(43a^2+12b^2)\,(e\sin(c+dx))^{7/2}}{231de} + \frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de}$$

output
```
-2/15*a*(3*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+2/231*b*(43*a^2+
12*b^2)*(e*sin(d*x+c))^(7/2)/d/e+10/33*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))
^(7/2)/d/e+2/11*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2)/d/e-2/5*a*(3*a^2
+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*
x+c)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx =$$
$$-\frac{(e\sin(c+dx))^{5/2}\left(1848(3a^3+2ab^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+(462a(4a^2+b^2)\cos(c+dx)+5b(-396a^2\right.}{4620d\sin^{\frac{5}{2}}(}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/4620*((e*Sin[c + d*x])^(5/2)*(1848*(3*a^3 + 2*a*b^2)*EllipticE[(-2*c +
Pi - 2*d*x)/4, 2] + (462*a*(4*a^2 + b^2)*Cos[c + d*x] + 5*b*(-396*a^2 - 69
*b^2 + 12*(33*a^2 + 4*b^2)*Cos[2*(c + d*x)] + 154*a*b*Cos[3*(c + d*x)] + 2
1*b^2*Cos[4*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5/2))
```

3.50.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \; 3171$$

$$\frac{2}{11}\int \frac{1}{2}(a+b\cos(c+dx))\left(11a^2+15b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}\int (a+b\cos(c+dx))\left(11a^2+15b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{5/2}dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\downarrow$ 3042

$$\frac{1}{11}\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(11a^2+15b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\downarrow$ 3341

$$\frac{1}{11}\left(\frac{2}{9}\int \frac{3}{2}\left(11a\left(3a^2+2b^2\right)+b\left(43a^2+12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{5/2}dx+\frac{10ab(e\sin(c+dx))^{7/2}(a+b\cos}{3de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{3}\int \left(11a\left(3a^2+2b^2\right)+b\left(43a^2+12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{5/2}dx+\frac{10ab(e\sin(c+dx))^{7/2}(a+b\cos(c}{3de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{3}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(11a\left(3a^2+2b^2\right)-b\left(43a^2+12b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{10ab(e\sin(c+dx))}{3}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\downarrow$ 3148

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\int (e\sin(c+dx))^{5/2}dx+\frac{2b\left(43a^2+12b^2\right)(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{10ab(e\sin(c+dx))^{7/2}(a}{3de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\downarrow$ 3042

3.50. $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx$

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\int(e\sin(c+dx))^{5/2}dx+\frac{2b\left(43a^2+12b^2\right)(e\sin(c+dx))^{7/2}}{7de}\right)+\frac{10ab(e\sin(c+dx))^{7/2}(a+\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3115

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3}{5}e^2\int\sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b\left(43a^2+12b^2\right)(e\sin(\right.}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3}{5}e^2\int\sqrt{e\sin(c+dx)}dx-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b\left(43a^2+12b^2\right)(e\sin(\right.}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3121

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(43a^2+}{}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{3e^2\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{5\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(43a^2+}{}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

$\big\downarrow$ 3119

$$\frac{1}{11}\left(\frac{1}{3}\left(11a\left(3a^2+2b^2\right)\left(\frac{6e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}\right)+\frac{2b(43a^2}{}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}$$

input
```
Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

3.50. $\int(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx$

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2))/(11*d*e) + ((10*a*b*(a
 + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(3*d*e) + ((2*b*(43*a^2 + 12*b^
2)*(e*Sin[c + d*x])^(7/2))/(7*d*e) + 11*a*(3*a^2 + 2*b^2)*((6*e^2*Elliptic
E[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) -
(2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)))/3)/11
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

3.50.4 Maple [A] (verified)

Time = 40.31 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.76

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}\left(7b^2\left(\cos^2(dx+c)\right)+33a^2+4b^2\right)}{77e}-\frac{e^3a\left(10\left(\sin^6(dx+c)\right)b^2+18\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right.\right.}{}}{}$
parts	$-\frac{a^3e^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right.\right.}{5\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/77/e*b*(e*sin(d*x+c))^(7/2)*(7*b^2*cos(d*x+c)^2+33*a^2+4*b^2)-1/15*e^3*
a*(10*sin(d*x+c)^6*b^2+18*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+12*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c)
)^(1/2),1/2*2^(1/2))*b^2-9*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x
+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c)
)^(1/2),1/2*2^(1/2))*b^2-6*a^2*sin(d*x+c)^4-14*sin(d*x+c)^4*b^2+6*a^2*sin(
d*x+c)^2+4*b^2*sin(d*x+c)^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.50.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \frac{231i\sqrt{2}(3a^3+2ab^2)\sqrt{-i\,e}e^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\text{s}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/1155*(231*I*sqrt(2)*(3*a^3 + 2*a*b^2)*sqrt(-I*e)*e^2*weierstrassZeta(4,
0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(
2)*(3*a^3 + 2*a*b^2)*sqrt(I*e)*e^2*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(105*b^3*e^2*cos(d*x + c)^4 +
 385*a*b^2*e^2*cos(d*x + c)^3 + 45*(11*a^2*b - b^3)*e^2*cos(d*x + c)^2 + 2
31*(a^3 - a*b^2)*e^2*cos(d*x + c) - 15*(33*a^2*b + 4*b^3)*e^2)*sqrt(e*sin(
d*x + c))*sin(d*x + c))/d
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.50.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2), x)
```

3.50.8 Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \int (e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

3.51 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx$

3.51.1 Optimal result

Integrand size = 25, antiderivative size = 202

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{2a(7a^2+6b^2)\,e^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de} + \frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de} + \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de}$$

output
```
2/315*b*(89*a^2+28*b^2)*(e*sin(d*x+c))^(5/2)/d/e+26/63*a*b*(a+b*cos(d*x+c)
)*(e*sin(d*x+c))^(5/2)/d/e+2/9*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2)/d
/e-2/21*a*(7*a^2+6*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1
/2)/d/(e*sin(d*x+c))^(1/2)-2/21*a*(7*a^2+6*b^2)*e*cos(d*x+c)*(e*sin(d*x+c)
)^(1/2)/d
```

3.51.2 Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{\left(-20a(28a^2+15b^2)\cot(c+dx)-\frac{2}{3}b(-756a^2-147b^2+28(27a^2+4b^2)\cos(2(c+dx))+270\right.}{}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

output
```
((-20*a*(28*a^2 + 15*b^2)*Cot[c + d*x] - (2*b*(-756*a^2 - 147*b^2 + 28*(27
*a^2 + 4*b^2)*Cos[2*(c + d*x)] + 270*a*b*Cos[3*(c + d*x)] + 35*b^2*Cos[4*(
c + d*x)])*Csc[c + d*x])/3 - (80*a*(7*a^2 + 6*b^2)*EllipticF[(-2*c + Pi -
2*d*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(840*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{9}\int \frac{1}{2}(a+b\cos(c+dx))\left(9a^2+13b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int (a+b\cos(c+dx))\left(9a^2+13b\cos(c+dx)a+4b^2\right)(e\sin(c+dx))^{3/2}dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(9a^2+13b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3341}$$

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{1}{2}\left(9a\left(7a^2+6b^2\right)+b\left(89a^2+28b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}dx+\frac{26ab(e\sin(c+dx))^{5/2}(a+b\cos(c\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \left(9a\left(7a^2+6b^2\right)+b\left(89a^2+28b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}dx+\frac{26ab(e\sin(c+dx))^{5/2}(a+b\cos(c+\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\int \left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(9a\left(7a^2+6b^2\right)-b\left(89a^2+28b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{26ab(e\sin(c+dx))^{5/2}}{7d}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\int (e\sin(c+dx))^{3/2}dx+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{26ab(e\sin(c+dx))^{5/2}(a+b}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$$\Big\downarrow \text{3042}$$

3.51. $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx$

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\int (e\sin(c+dx))^{3/2}dx+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+dx))^{5/2}}{5de}\right)+\frac{26ab(e\sin(c+dx))^{5/2}(a+b}{7de}\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\big\downarrow$ 3115

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{1}{3}e^2\int\frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e\sin(c+}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\big\downarrow$ 3121

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e\sin}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{e^2\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{3\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b\left(89a^2+28b^2\right)(e\sin}{5de}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

$\big\downarrow$ 3120

$$\frac{1}{9}\left(\frac{1}{7}\left(9a\left(7a^2+6b^2\right)\left(\frac{2e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}\right)+\frac{2b(89a}{}\right.\right.$$
$$\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}$$

3.51. $\int(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx$

input
```
Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2))/(9*d*e) + ((26*a*b*(a
+ b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(7*d*e) + ((2*b*(89*a^2 + 28*b^2
)*(e*Sin[c + d*x])^(5/2))/(5*d*e) + 9*a*(7*a^2 + 6*b^2)*((2*e^2*EllipticF[
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2
*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)))/7)/9
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

3.51.4 Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.35

method	result
parts	$-\frac{a^{3} e^{2} \left(\sqrt{1-\sin \left(d x +c \right)}\, \sqrt{2 \sin \left(d x +c \right)+2}\, \left(\sqrt{\sin}\left(d x +c \right)\right) F\left(\sqrt{1-\sin \left(d x +c \right)}, \frac{\sqrt{2}}{2}\right)-2 \left(\sin^{3}\left(d x +c \right)\right)+2 \sin \left(d x +c \right)\right)}{3 \cos \left(d x +c \right) \sqrt{e \sin \left(d x +c \right)}\, d}-\frac{2 b^{3} \left(\frac{\left(e \sin (d x +}{9}\right.}{}$
default	$-\frac{e^{2} \left(70 b^{3} \left(\cos^{5}\left(d x +c \right)\right) \sin \left(d x +c \right)+270 a\, b^{2} \left(\cos^{4}\left(d x +c \right)\right) \sin \left(d x +c \right)+105 \sqrt{1-\sin \left(d x +c \right)}\, \sqrt{2 \sin \left(d x +c \right)+2}\, \left(\sqrt{\sin}\left(d x +c \right)\right) F\left(\sqrt{1-\sin}\right.\right.}{}$

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3*e^2*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/
cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2*b^3/d/e^3*(1/9*(e*sin(d*x+c))^(9/2)-1/
5*e^2*(e*sin(d*x+c))^(5/2))+6/5*a^2*b*(e*sin(d*x+c))^(5/2)/e/d-2/7*a*b^2*e
^2*(3*sin(d*x+c)^5+(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x
+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d
```

3.51.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \frac{15\sqrt{2}(7a^3+6ab^2)\sqrt{-ie}\,e\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+15\sqrt{2}(7a}{}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/315*(15*sqrt(2)*(7*a^3 + 6*a*b^2)*sqrt(-I*e)*e*weierstrassPInverse(4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(7*a^3 + 6*a*b^2)*sqrt(I*e)*e
*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(35*b^3*e*co
s(d*x + c)^4 + 135*a*b^2*e*cos(d*x + c)^3 + 7*(27*a^2*b - b^3)*e*cos(d*x +
 c)^2 + 15*(7*a^3 - 3*a*b^2)*e*cos(d*x + c) - 7*(27*a^2*b + 4*b^3)*e)*sqrt
(e*sin(d*x + c)))/d
```

3.51.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (e\sin(c+dx))^{\frac{3}{2}}(a+b\cos(c+dx))^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**3, x)
```

3.51.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2), x)
```

3.51.8 Giac [F]

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx = \int (e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

3.52 $\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx$

3.52.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx &= \frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{2b(57a^2+20b^2)\,(e\sin(c+dx))^{3/2}}{105de}\\ &\quad+\frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de}+\frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de}\end{aligned}$$

output
```
2/105*b*(57*a^2+20*b^2)*(e*sin(d*x+c))^(3/2)/d/e+22/35*a*b*(a+b*cos(d*x+c)
)*(e*sin(d*x+c))^(3/2)/d/e+2/7*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2)/d
/e-2/5*a*(5*a^2+6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/
2)/d/sin(d*x+c)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \frac{\sqrt{e\sin(c+dx)}\left(-42(5a^3+6ab^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b(210a^2+55b^2+126ab\cos(c+dx)+15b^2\right.}{105d\sqrt{\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(Sqrt[e*Sin[c + d*x]]*(-42*(5*a^3 + 6*a*b^2)*EllipticE[(-2*c + Pi - 2*d*x)
/4, 2] + b*(210*a^2 + 55*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*
x)])*Sin[c + d*x]^(3/2)))/(105*d*Sqrt[Sin[c + d*x]])
```

3.52.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\cos(c+dx))\left(7a^2+11b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int (a+b\cos(c+dx))\left(7a^2+11b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(7a^2+11b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$$\Big\downarrow \text{3341}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\left(7a\left(5a^2+6b^2\right)+b\left(57a^2+20b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\cos(c+\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \left(7a\left(5a^2+6b^2\right)+b\left(57a^2+20b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\cos(c+d\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(7a\left(5a^2+6b^2\right)-b\left(57a^2+20b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{22ab(e\sin(c+dx))^{3/2}(a\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\Big\downarrow$ 3148

$$\frac{1}{7}\left(\frac{1}{5}\left(7a\left(5a^2+6b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\,c\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(7a\left(5a^2+6b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\,c\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\Big\downarrow$ 3121

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{7a\left(5a^2+6b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(57a^2+20b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+d\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\Big\downarrow$ 3042

3.52. $\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{7a(5a^2+6b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(57a^2+20b^2)(e\sin(c+dx))^{3/2}}{3de}\right)+\frac{22ab(e\sin(c+d\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

$\big\downarrow$ 3119

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2b(57a^2+20b^2)(e\sin(c+dx))^{3/2}}{3de}+\frac{14a(5a^2+6b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}\right)+\frac{22ab(e\sin(c\right.$$
$$\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}$$

input
```
Int[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2))/(7*d*e) + ((22*a*b*(a
+ b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(5*d*e) + ((14*a*(5*a^2 + 6*b^2)
*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*
x]]) + (2*b*(57*a^2 + 20*b^2)*(e*Sin[c + d*x])^(3/2))/(3*d*e))/5)/7
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

3.52.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.95

method	result
parts	$-\frac{a^{3} e \sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^{3}\left(\frac{(e\sin(dx+c))^{\frac{7}{2}}}{7}\right.}{d}$
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}\left(3b^{2}\left(\cos^{2}(dx+c)\right)+21a^{2}+4b^{2}\right)}{21e}-\frac{ae\left(10\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^{2}+12\sqrt{1-\sin(dx+c)}\right.}{}}{}$

input
```
int((a+cos(d*x+c)*b)^3*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^3*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(2*Ell
ipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/
2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d-2*b^3/d/e^3*(1/7*(e*sin(d*x+
c))^(7/2)-1/3*e^2*(e*sin(d*x+c))^(3/2))-6/5*a*b^2*e*(2*(1-sin(d*x+c))^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1
/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)^4-cos(d*x+c)^2)/cos(
d*x+c)/(e*sin(d*x+c))^(1/2)/d+2*a^2*b*(e*sin(d*x+c))^(3/2)/e/d
```

3.52.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx =$$
$$-\frac{21\sqrt{2}(-5i\,a^3-6i\,ab^2)\sqrt{-i\,e}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(dx+c)}+i\,\sin{(dx+c}$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(21*sqrt(2)*(-5*I*a^3 - 6*I*a*b^2)*sqrt(-I*e)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(5
*I*a^3 + 6*I*a*b^2)*sqrt(I*e)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*b^3*cos(d*x + c)^2 + 63*a*b^2*
cos(d*x + c) + 105*a^2*b + 20*b^3)*sqrt(e*sin(d*x + c))*sin(d*x + c))/d
```

3.52.6 Sympy [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \int \sqrt{e\sin{(c+dx)}}(a+b\cos{(c+dx)})^3\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**3, x)
```

3.52.7 Maxima [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx=\int (b\cos(dx+c)+a)^3\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c)), x)
```

3.52.8 Giac [F]

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx=\int (b\cos(dx+c)+a)^3\sqrt{e\sin(dx+c)}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c)), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx=\int \sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))^3\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

3.53 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$

3.53.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx &= \frac{2a(a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}\\ &\quad+\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de}\\ &\quad+\frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de}\\ &\quad+\frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de}\end{aligned}$$

output
```
-2*a*(a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*si
n(d*x+c))^(1/2)+2/5*b*(11*a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e+6/5*a*b*(a+b
*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/d/e+2/5*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+
c))^(1/2)/d/e
```

3.53.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \frac{-10a(a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right)\sqrt{\sin(c+dx)}+b(30a^2+9b^2+10ab\cos(c+dx)+b^2\cos}{5d\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(-10*a*(a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]
] + b*(30*a^2 + 9*b^2 + 10*a*b*Cos[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c
+ d*x])/(5*d*Sqrt[e*Sin[c + d*x]])
```

3.53.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3171, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3171$$

$$\frac{2}{5}\int \frac{(a+b\cos(c+dx))\left(5a^2+9b\cos(c+dx)a+4b^2\right)}{2\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\int \frac{(a+b\cos(c+dx))\left(5a^2+9b\cos(c+dx)a+4b^2\right)}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)\left(5a^2-9b\sin\left(c+dx-\frac{\pi}{2}\right)a+4b^2\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{3341}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3\left(5a\left(a^2+2b^2\right)+b\left(11a^2+4b^2\right)\cos(c+dx)\right)}{2\sqrt{e\sin(c+dx)}}dx+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\int \frac{5a\left(a^2+2b^2\right)+b\left(11a^2+4b^2\right)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\int \frac{5a\left(a^2+2b^2\right)-b\left(11a^2+4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{5}\left(5a\left(a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\left(11a^2+4b^2\right)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(5a\left(a^2+2b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx+\frac{2b\left(11a^2+4b^2\right)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}\right)+\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$$\Big\downarrow \text{3121}$$

3.53. $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$

$$\frac{1}{5}\left(\frac{5a\left(a^2+2b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b\left(11a^2+4b^2\right)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c}{de}\right.$$
$$\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{5a\left(a^2+2b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}+\frac{2b\left(11a^2+4b^2\right)\sqrt{e\sin(c+dx)}}{de}+\frac{6ab\sqrt{e\sin(c+dx)}(a+b\cos(c}{de}\right.$$
$$\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{2b\left(11a^2+4b^2\right)\sqrt{e\sin(c+dx)}}{de}+\frac{10a\left(a^2+2b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{6ab\sqrt{e\sin(c+}{}\right.$$
$$\frac{2b\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}{5de}$$

input
```
Int[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]
```

output
```
(2*b*(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]])/(5*d*e) + ((10*a*(a^2 +
2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[
c + d*x]]) + (2*b*(11*a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]])/(d*e) + (6*a*b*(a
 + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(d*e))/5
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

3.53.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.34

method	result
default	$-\frac{5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^3+10\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F}{5\cos(d}$
parts	$-\frac{a^3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^3\left(\frac{(e\sin(dx+c))^{\frac{5}{2}}}{5}-\sqrt{e\sin(dx+c)}\,e^2\right)}{d\,e^3}+\frac{3a\,b^2}{}$

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^3
+10*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-2*b^3*cos(d*x+c)^3*sin(d*x+c)-10*
a*b^2*cos(d*x+c)^2*sin(d*x+c)-30*a^2*b*cos(d*x+c)*sin(d*x+c)-8*b^3*cos(d*x
+c)*sin(d*x+c))/d
```

3.53.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{5\sqrt{2}(a^3+2\,ab^2)\sqrt{-i\,e}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(a^3+2\,ab^2)\sqrt{i\,e}\text{weie}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(5*sqrt(2)*(a^3 + 2*a*b^2)*sqrt(-I*e)*weierstrassPInverse(4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(a^3 + 2*a*b^2)*sqrt(I*e)*weierstrass
PInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(b^3*cos(d*x + c)^2 + 5*
a*b^2*cos(d*x + c) + 15*a^2*b + 4*b^3)*sqrt(e*sin(d*x + c)))/(d*e)
```

3.53.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3/sqrt(e*sin(c + d*x)), x)
```

3.53.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sqrt(e*sin(d*x + c)), x)
```

3.53.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/sqrt(e*sin(d*x + c)), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{\sqrt{e\,\sin(c+d\,x)}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2), x)
```

3.54 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx$

3.54.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a(a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} - \frac{2b(3a^2+4b^2)\,(e\sin(c+dx))^{3/2}}{3de^3} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{de^3}$$

output
```
-2/3*b*(3*a^2+4*b^2)*(e*sin(d*x+c))^(3/2)/d/e^3-2*a*b*(a+b*cos(d*x+c))*(e*
sin(d*x+c))^(3/2)/d/e^3-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d
*x+c))^(1/2)+2*a*(a^2+6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c
))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2\left(9a^2b+3b^3+3a(a^2+3b^2)\cos(c+dx)-3a(a^2+6b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\sqrt{\sin(c+dx)}+b^3\sin^2(\right.}{3de\sqrt{e\sin(c+dx)}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(9*a^2*b + 3*b^3 + 3*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*a*(a^2 + 6*b^2)*
EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]] + b^3*Sin[c + d*x]^
2))/(3*d*e*Sqrt[e*Sin[c + d*x]])
```

3.54.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3170, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3170}$$

$$-\frac{2\int \frac{1}{2}(a+b\cos(c+dx))\left(a^2+5b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int (a+b\cos(c+dx))\left(a^2+5b\cos(c+dx)a+4b^2\right)\sqrt{e\sin(c+dx)}dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a^2+5b\sin\left(c+dx+\frac{\pi}{2}\right)a+4b^2\right)dx}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow \text{ 3341}$$

$$-\frac{\frac{\frac{2}{5}\int \frac{5}{2}\left(a\left(a^2+6b^2\right)+b\left(3a^2+4b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}}{}$$

$$\Big\downarrow 27$$

$$-\frac{\int \left(a\left(a^2+6b^2\right)+b\left(3a^2+4b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow 3042$$

$$-\frac{\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a\left(a^2+6b^2\right)-b\left(3a^2+4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow 3148$$

$$-\frac{a\left(a^2+6b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(3a^2+4b^2\right)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow 3042$$

$$-\frac{a\left(a^2+6b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(3a^2+4b^2\right)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow 3121$$

$$-\frac{\frac{a\left(a^2+6b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(3a^2+4b^2\right)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

$$\Big\downarrow 3042$$

$$-\frac{\frac{a\left(a^2+6b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(3a^2+4b^2\right)(e\sin(c+dx))^{3/2}}{3de}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

3.54. $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{3119}$$

$$-\frac{\frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de}+\frac{2a(a^2+6b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de}}{e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(d*e*Sqrt[e*Sin[c + d*x]]
) - ((2*a*(a^2 + 6*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(d*Sqrt[Sin[c + d*x]]) + (2*b*(3*a^2 + 4*b^2)*(e*Sin[c + d*x])^(3/2))
/(3*d*e) + (2*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(d*e))/e^2
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

3.54.4 Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.90

method	result
default	$-\frac{3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^{3}+18\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F}{}$
parts	$\frac{a^{3}\left(2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)F\left(\sqrt{}\right.\right.}{e\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*(3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a
^3+18*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-6*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2)
)*a^3-36*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a*b^2-2*b^3*cos(d*x+c)^3+6*a^3*cos
(d*x+c)^2+18*a*b^2*cos(d*x+c)^2+18*a^2*b*cos(d*x+c)+8*b^3*cos(d*x+c))/d
```

3.54.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx =$$
$$-\frac{3\sqrt{2}(i\,a^3+6i\,ab^2)\sqrt{-i\,e}\sin\left(dx+c\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(d\right.$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(3*sqrt(2)*(I*a^3 + 6*I*a*b^2)*sqrt(-I*e)*sin(d*x + c)*weierstrassZet
a(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt
(2)*(-I*a^3 - 6*I*a*b^2)*sqrt(I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(b^3*cos(d*x + c
)^2 - 9*a^2*b - 4*b^3 - 3*(a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c
)))/(d*e^2*sin(d*x + c))
```

3.54.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cos\left(c+dx\right))^3}{(e\sin\left(c+dx\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(3/2), x)
```

3.54.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos\left(dx+c\right)+a)^3}{(e\sin\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(3/2), x)
```

3.54.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(3/2), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{(e\,\sin(c+d\,x))^{3/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2), x)
```

3.55 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx$

3.55.1 Optimal result

Integrand size = 25, antiderivative size = 169

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx &= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} \\ &\quad+\frac{2a(a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} \\ &\quad-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3}-\frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de^3}\end{aligned}$$

output
```
-2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(a
^2-6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d
*x+c))^(1/2)-2/3*b*(a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e^3-2/3*a*b*(a+b*cos
(d*x+c))*(e*sin(d*x+c))^(1/2)/d/e^3
```

3.55.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx =$$

$$-\frac{6a^2b+5b^3+2a(a^2+3b^2)\cos(c+dx)-3b^3\cos(2(c+dx))+2a(a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{4}(-2c+\pi-2dx),2\right.}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

output
```
-1/3*(6*a^2*b + 5*b^3 + 2*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*b^3*Cos[2*(c +
d*x)] + 2*a*(a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]
^(3/2))/(d*e*(e*Sin[c + d*x])^(3/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3170, 27, 3042, 3341, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{2\int -\frac{(a+b\cos(c+dx))\left(a^2-3b\cos(c+dx)a-4b^2\right)}{2\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\cos(c+dx))\left(a^2-3b\cos(c+dx)a-4b^2\right)}{\sqrt{e\sin(c+dx)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)\left(a^2+3b\sin\left(c+dx-\frac{\pi}{2}\right)a-4b^2\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3341}$$

$$\frac{\frac{2}{3}\int \frac{3\left(a\left(a^2-6b^2\right)-b\left(a^2+4b^2\right)\cos(c+dx)\right)}{2\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a(a^2-6b^2)-b(a^2+4b^2)\cos(c+dx)}{\sqrt{e\sin(c+dx)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(a^2-6b^2)+b(a^2+4b^2)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}dx-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3148$$

$$\frac{a(a^2-6b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(a^2-6b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{a(a^2-6b^2)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a(a^2-6b^2)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3120$$

3.55. $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{de}+\frac{2a(a^2-6b^2)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}{de}}{3e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(3*d*e*(e*Sin[c + d*x])^(
3/2)) + ((2*a*(a^2 - 6*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - (2*b*(a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]])
/(d*e) - (2*a*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(d*e))/(3*e^2)
```

3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

3.55.4 Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.34

method	result
default	$\frac{-\frac{2b\left(-3b^2\left(\cos^2(dx+c)\right)+3a^2+4b^2\right)}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-6b^2\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+}\right.}{3e^2\sin(dx+c)^2\cos(}}{d}$
parts	$-\frac{a^3\left(\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(dx+c)\right)+2\sin(dx+c)\right)}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-\frac{2b^3\left(\sqrt{e\sin(dx+}\right.}{}$

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/3*b/e/(e*sin(d*x+c))^(3/2)*(-3*b^2*cos(d*x+c)^2+3*a^2+4*b^2)-1/3*a/e^2
*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+
2*a^2*cos(d*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/
cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.55.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}(a^3-6\,ab^2)\cos{(dx+c)}^2-\sqrt{2}(a^3-6\,ab^2)\right)\sqrt{-i\,e}\,\mathrm{weierstrassPInverse}(4,0$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*((sqrt(2)*(a^3 - 6*a*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^3 - 6*a*b^2))*sq
rt(-I*e)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(
2)*(a^3 - 6*a*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^3 - 6*a*b^2))*sqrt(I*e)*wei
erstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(3*b^3*cos(d*x +
 c)^2 - 3*a^2*b - 4*b^3 - (a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c
)))/(d*e^3*cos(d*x + c)^2 - d*e^3)
```

3.55.6 Sympy [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cos{(c+dx)})^3}{(e\sin{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(5/2), x)
```

3.55.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos{(dx+c)}+a)^3}{(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(5/2), x)
```

3.55.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(5/2), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(a+b\,\cos(c+d\,x))^3}{(e\,\sin(c+d\,x))^{5/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2), x)
```

3.56 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx$

3.56.1 Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{6a(a^2-2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}} - \frac{2b(3a^2-4b^2)\,(e\sin(c+dx))^{3/2}}{5de^5}$$

output
```
-2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(5/2)-2/5*b*(3
*a^2-4*b^2)*(e*sin(d*x+c))^(3/2)/d/e^5+2/5*(a+b*cos(d*x+c))*(a*b-(3*a^2-4*
b^2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(1/2)+6/5*a*(a^2-2*b^2)*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{12a^2b-6b^3+a(7a^2+6b^2)\cos(c+dx)+10b^3\cos(2(c+dx))-3a^3\cos(3(c+dx))+6ab^2\cos(3(c+dx))}{10de(e\sin(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]
```

output
```
-1/10*(12*a^2*b - 6*b^3 + a*(7*a^2 + 6*b^2)*Cos[c + d*x] + 10*b^3*Cos[2*(c
 + d*x)] - 3*a^3*Cos[3*(c + d*x)] + 6*a*b^2*Cos[3*(c + d*x)] - 12*a*(a^2 -
 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(d*e*(e*Si
n[c + d*x])^(5/2))
```

3.56.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3170, 27, 3042, 3340, 27, 3042, 3148, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{2\int -\frac{(a+b\cos(c+dx))(3a^2-b\cos(c+dx)a-4b^2)}{2(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\cos(c+dx))(3a^2-b\cos(c+dx)a-4b^2)}{(e\sin(c+dx))^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)\left(3a^2+b\sin\left(c+dx-\frac{\pi}{2}\right)a-4b^2\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}dx}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3340}$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{3}{2}\left(a\left(a^2-2b^2\right)+b\left(3a^2-4b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\int \left(a\left(a^2-2b^2\right)+b\left(3a^2-4b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}dx}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\int \sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a\left(a^2-2b^2\right)-b\left(3a^2-4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3148$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(a\left(a^2-2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(3a^2-4b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(a\left(a^2-2b^2\right)\int \sqrt{e\sin(c+dx)}dx+\frac{2b\left(3a^2-4b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3121$$

$$\frac{\frac{2(a+b\cos(c+dx))\left(ab-\left(3a^2-4b^2\right)\cos(c+dx)\right)}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(\frac{a\left(a^2-2b^2\right)\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b\left(3a^2-4b^2\right)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

3.56. $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx$

$$\frac{\frac{2(a+b\cos(c+dx))(ab-(3a^2-4b^2)\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(\frac{a(a^2-2b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+\frac{2b(3a^2-4b^2)(e\sin(c+dx))^{3/2}}{3de}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{2(a+b\cos(c+dx))(ab-(3a^2-4b^2)\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{3\left(\frac{2b(3a^2-4b^2)(e\sin(c+dx))^{3/2}}{3de}+\frac{2a(a^2-2b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}\right)}{e^2}}{5e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(5*d*e*(e*Sin[c + d*x])^(
5/2)) + ((2*(a + b*Cos[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Cos[c + d*x]))/(d*
e*Sqrt[e*Sin[c + d*x]]) - (3*((2*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x
)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (2*b*(3*a^2 - 4*b^2
)*(e*Sin[c + d*x])^(3/2))/(3*d*e)))/e^2)/(5*e^2)
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

3.56.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.95

method	result
default	$\frac{-\frac{2b\left(5b^2\left(\cos^2(dx+c)\right)+3a^2-4b^2\right)}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^2-12\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c}}{}}{}$
parts	$\frac{a^3\left(6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)E\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)F\left(\right.}{5e^3\sin(dx+c)^3\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/5*b/e/(e*sin(d*x+c))^(5/2)*(5*b^2*cos(d*x+c)^2+3*a^2-4*b^2)+1/5*a/e^3*
(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^
2-3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF
((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^
2+6*a^2*cos(d*x+c)^4*sin(d*x+c)-12*b^2*cos(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d
*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^3/cos(d*x+c)/
(e*sin(d*x+c))^(1/2))/d
```

3.56.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx =$$
$$-\frac{3\left(\sqrt{2}(i\,a^3-2i\,ab^2)\cos\left(dx+c\right)^2+\sqrt{2}(-i\,a^3+2i\,ab^2)\right)\sqrt{-i\,e}\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstras}}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(sqrt(2)*(I*a^3 - 2*I*a*b^2)*cos(d*x + c)^2 + sqrt(2)*(-I*a^3 + 2*
I*a*b^2))*sqrt(-I*e)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(-I*a^3 + 2*I*a*b^2)*
cos(d*x + c)^2 + sqrt(2)*(I*a^3 - 2*I*a*b^2))*sqrt(I*e)*sin(d*x + c)*weier
strassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*(5*b^3*cos(d*x + c)^2 - 3*(a^3 - 2*a*b^2)*cos(d*x + c)^3 + 3*a^2*b -
4*b^3 + (4*a^3 - 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/((d*e^4*cos(
d*x + c)^2 - d*e^4)*sin(d*x + c))
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(7/2), x)
```

3.56.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(7/2), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = \int \frac{(a+b\,\cos\left(c+d\,x\right))^3}{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2), x)
```

3.57 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx$

3.57.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}} + \frac{2a(5a^2-6b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21de^4\sqrt{e\sin(c+dx)}} - \frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5}$$

output
```
-2/7*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(7/2)-2/21*(a+
b*cos(d*x+c))*(a*b+(5*a^2-4*b^2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(3/2)-2/
21*a*(5*a^2-6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^4
/(e*sin(d*x+c))^(1/2)-2/21*b*(5*a^2-4*b^2)*(e*sin(d*x+c))^(1/2)/d/e^5
```

3.57.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx =$$
$$-\frac{2\csc^4(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{1}{4}(36a^2b-2b^3+a(17a^2+30b^2)\cos(c+dx)+14b^3\cos(2(c+dx))-5a^3\cos\right.}{21de^5}$$

input
```
Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

output
```
(-2*Csc[c + d*x]^4*Sqrt[e*Sin[c + d*x]]*((36*a^2*b - 2*b^3 + a*(17*a^2 + 3
0*b^2)*Cos[c + d*x] + 14*b^3*Cos[2*(c + d*x)] - 5*a^3*Cos[3*(c + d*x)] + 6
*a*b^2*Cos[3*(c + d*x)])/4 + a*(5*a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*
x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*d*e^5)
```

3.57.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3170, 27, 3042, 3340, 27, 3042, 3148, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{2\int -\frac{(a+b\cos(c+dx))(5a^2+b\cos(c+dx)a-4b^2)}{2(e\sin(c+dx))^{5/2}}dx}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\cos(c+dx))(5a^2+b\cos(c+dx)a-4b^2)}{(e\sin(c+dx))^{5/2}}dx}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(a-b \sin \left(c+d x-\frac{\pi}{2}\right)\right)\left(5 a^2-b \sin \left(c+d x-\frac{\pi}{2}\right) a-4 b^2\right)}{\left(e \cos \left(c+d x-\frac{\pi}{2}\right)\right)^{5/2}}dx}{7 e^2}-\frac{2 (a \cos (c+d x)+b) (a+b \cos (c+d x))^2}{7 d e (e \sin (c+d x))^{7/2}}$$

$$\Big\downarrow \text{3340}$$

$$\frac{-\frac{2 \int -\frac{a \left(5 a^2-6 b^2\right)-b \left(5 a^2-4 b^2\right) \cos (c+d x)}{2 \sqrt{e \sin (c+d x)}}dx}{3 e^2}-\frac{2 (a+b \cos (c+d x)) \left(\left(5 a^2-4 b^2\right) \cos (c+d x)+a b\right)}{3 d e (e \sin (c+d x))^{3/2}}}{7 e^2}-\frac{2 (a \cos (c+d x)+b) (a+b \cos (c+d x))^2}{7 d e (e \sin (c+d x))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a \left(5 a^2-6 b^2\right)-b \left(5 a^2-4 b^2\right) \cos (c+d x)}{\sqrt{e \sin (c+d x)}}dx}{3 e^2}-\frac{2 (a+b \cos (c+d x)) \left(\left(5 a^2-4 b^2\right) \cos (c+d x)+a b\right)}{3 d e (e \sin (c+d x))^{3/2}}}{7 e^2}-\frac{2 (a \cos (c+d x)+b) (a+b \cos (c+d x))^2}{7 d e (e \sin (c+d x))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a \left(5 a^2-6 b^2\right)+b \left(5 a^2-4 b^2\right) \sin \left(c+d x-\frac{\pi}{2}\right)}{\sqrt{e \cos \left(c+d x-\frac{\pi}{2}\right)}}dx}{3 e^2}-\frac{2 (a+b \cos (c+d x)) \left(\left(5 a^2-4 b^2\right) \cos (c+d x)+a b\right)}{3 d e (e \sin (c+d x))^{3/2}}}{7 e^2}-\frac{2 (a \cos (c+d x)+b) (a+b \cos (c+d x))^2}{7 d e (e \sin (c+d x))^{7/2}}$$

$$\Big\downarrow \text{3148}$$

$$\frac{\frac{a \left(5 a^2-6 b^2\right) \int \frac{1}{\sqrt{e \sin (c+d x)}}dx-\frac{2 b \left(5 a^2-4 b^2\right) \sqrt{e \sin (c+d x)}}{d e}}{3 e^2}-\frac{2 (a+b \cos (c+d x)) \left(\left(5 a^2-4 b^2\right) \cos (c+d x)+a b\right)}{3 d e (e \sin (c+d x))^{3/2}}}{7 e^2}-\frac{2 (a \cos (c+d x)+b) (a+b \cos (c+d x))^2}{7 d e (e \sin (c+d x))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a \left(5 a^2-6 b^2\right) \int \frac{1}{\sqrt{e \sin (c+d x)}}dx-\frac{2 b \left(5 a^2-4 b^2\right) \sqrt{e \sin (c+d x)}}{d e}}{3 e^2}-\frac{2 (a+b \cos (c+d x)) \left(\left(5 a^2-4 b^2\right) \cos (c+d x)+a b\right)}{3 d e (e \sin (c+d x))^{3/2}}}{7 e^2}-\frac{2 (a \cos (c+d x)+b) (a+b \cos (c+d x))^2}{7 d e (e \sin (c+d x))^{7/2}}$$

$$\Big\downarrow \text{3121}$$

3.57. $\int \frac{(a+b \cos (c+d x))^3}{(e \sin (c+d x))^{9/2}} \, dx$

$$\frac{\frac{\frac{a\left(5a^2-6b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b\left(5a^2-4b^2\right)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))\left(\left(5a^2-4b^2\right)\cos(c+dx)+ab\right)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a\left(5a^2-6b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-\frac{2b\left(5a^2-4b^2\right)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))\left(\left(5a^2-4b^2\right)\cos(c+dx)+ab\right)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\frac{2a\left(5a^2-6b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-\frac{2b\left(5a^2-4b^2\right)\sqrt{e\sin(c+dx)}}{de}}{3e^2}-\frac{2(a+b\cos(c+dx))\left(\left(5a^2-4b^2\right)\cos(c+dx)+ab\right)}{3de(e\sin(c+dx))^{3/2}}}{7e^2}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

input
```
Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(7*d*e*(e*Sin[c + d*x])^(
7/2)) + ((-2*(a + b*Cos[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Cos[c + d*x]))/(3
*d*e*(e*Sin[c + d*x])^(3/2)) + ((2*a*(5*a^2 - 6*b^2)*EllipticF[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - (2*b*(5*a^2 - 4
*b^2)*Sqrt[e*Sin[c + d*x]])/(d*e))/(3*e^2))/(7*e^2)
```

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

3.57. $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx$

3.57.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.37

method	result
default	$\frac{-\frac{2b\left(7b^{2}\left(\cos^{2}(dx+c)\right)+9a^{2}-4b^{2}\right)}{21e(e\sin(dx+c))^{\frac{7}{2}}}-\frac{a\left(5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{9}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)a^{2}-6\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)}\right.}{}}{}$
parts	$-\frac{a^{3}\left(5\sqrt{1-\sin(dx+c)}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{9}{2}}(dx+c)\right)F\left(\sqrt{1-\sin(dx+c)},\frac{\sqrt{2}}{2}\right)-10\left(\sin^{5}(dx+c)\right)+4\left(\sin^{3}(dx+c)\right)+6\sin(dx+c)\right)}{21e^{4}\sin(dx+c)^{4}\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}-$

input
```
int((a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/21*b/e/(e*sin(d*x+c))^(7/2)*(7*b^2*cos(d*x+c)^2+9*a^2-4*b^2)-1/21*a/e^
4*(5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(9/2)*Elliptic
F((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b
^2-10*a^2*cos(d*x+c)^4*sin(d*x+c)+12*b^2*cos(d*x+c)^4*sin(d*x+c)+16*a^2*co
s(d*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^4/cos(d*x+
c)/(e*sin(d*x+c))^(1/2))/d
```

3.57.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.53

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \frac{\left(\sqrt{2}(5\,a^3-6\,ab^2)\cos\left(dx+c\right)^4-2\,\sqrt{2}(5\,a^3-6\,ab^2)\cos\left(dx+c\right)^2+\sqrt{2}(5\,a^3-\right.}{}$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/21*((sqrt(2)*(5*a^3 - 6*a*b^2)*cos(d*x + c)^4 - 2*sqrt(2)*(5*a^3 - 6*a*b
^2)*cos(d*x + c)^2 + sqrt(2)*(5*a^3 - 6*a*b^2))*sqrt(-I*e)*weierstrassPInv
erse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(5*a^3 - 6*a*b^2)*cos
(d*x + c)^4 - 2*sqrt(2)*(5*a^3 - 6*a*b^2)*cos(d*x + c)^2 + sqrt(2)*(5*a^3
- 6*a*b^2))*sqrt(I*e)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
 c)) - 2*(7*b^3*cos(d*x + c)^2 - (5*a^3 - 6*a*b^2)*cos(d*x + c)^3 + 9*a^2*
b - 4*b^3 + (8*a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(e*sin(d*x + c)))/(d*e^5*c
os(d*x + c)^4 - 2*d*e^5*cos(d*x + c)^2 + d*e^5)
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.57.7 Maxima [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(9/2), x)
```

3.57.8 Giac [F]

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(b\cos(dx+c)+a)^3}{(e\sin(dx+c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c) + a)^3/(e*sin(d*x + c))^(9/2), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = \int \frac{(a+b\,\cos\left(c+d\,x\right))^3}{(e\,\sin\left(c+d\,x\right))^{9/2}}\,dx$$

input
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2),x)
```

output
```
int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2), x)
```

3.58 $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

3.58.1 Optimal result

Integrand size = 25, antiderivative size = 544

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx &= \frac{(-a^2+b^2)^{9/4}\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d}\\
&+\frac{(-a^2+b^2)^{9/4}\,e^{11/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{11/2}d}\\
&+\frac{2a(21a^4-49a^2b^2+33b^4)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{21b^6d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)^3\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)^3\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d}\\
&+\frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}\end{aligned}$$

output
```
(-a^2+b^2)^(9/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(11/2)/d+(-a^2+b^2)^(9/4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(
d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+2/35*e^3*(7*a^2-7*b^2-5
*a*b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d-2/9*e*(e*sin(d*x+c))^(9/2)/b/d
-2/21*a*(21*a^4-49*a^2*b^2+33*b^4)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^2-b
*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4
*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^2
-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-2/21*e^5*(21*(a^2-b^2)^2-a*b
*(7*a^2-12*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

3.58.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 48.77 (sec) , antiderivative size = 2035, normalized size of antiderivative = 3.74

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(((a*(28*a^2 - 51*b^2)*Cos[c + d*x])/(42*b^4) + ((-9*a^2 + 14*b^2)*Cos[2*(
c + d*x)])/(45*b^3) + (a*Cos[3*(c + d*x)])/(14*b^2) - Cos[4*(c + d*x)]/(36
*b))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*
((2*(392*a^3*b - 722*a*b^3)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2]
)*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4
)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
- Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2
])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2
)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c +
d*x]^2)) + (2*(-280*a^4 + 636*a^2*b^2 - 721*b^4)*Cos[c + d*x]*(a + b*Sqrt[
1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]
*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)...
```

3.58.3 Rubi [A] (warning: unable to verify)

Time = 2.83 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.02, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3174, 25, 3042, 3344, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{11/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$$\Big\downarrow \; 25$$

$$\frac{e^2\int \frac{(b+a\cos(c+dx))(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$$\Big\downarrow \; 3344$$

$$\frac{e^2\left(\frac{2e^2\int -\frac{\left(b\left(2a^2-7b^2\right)+a\left(7a^2-12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{2(a+b\cos(c+dx))}dx}{7b^2}+\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}\right)}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\int \frac{\left(b\left(2a^2-7b^2\right)+a\left(7a^2-12b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{7b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b\left(2a^2-7b^2\right)+a\left(7a^2-12b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{7b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$$\Big\downarrow \; 3344$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e^2\int -\frac{b\left(14a^4-30b^2a^2+21b^4\right)+a\left(21a^4-49b^2a^2+33b^4\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(\right.\right.}{}\right.}{7b^2}\right.}{b}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$\downarrow$ 27

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{b\left(14a^4-30b^2a^2+21b^4\right)+a\left(\right.}{(a+b\cos(c+d}}{3}\right.}{7b^2}\right.$$
$$\overline{\qquad b \qquad}$$
$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3042

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{b\left(14a^4-30b^2a^2+21b^4\right)-a\left(\right.}{\sqrt{e\cos\left(c+dx-\right.}}}{}\right.}{7b^2}\right.$$
$$\overline{\qquad b \qquad}$$
$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3346

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\int\frac{}{\sqrt{}}}{b}\right.}{}\right.}{7b^2}\right.$$
$$\overline{\qquad b \qquad}$$
$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3042

3.58. $\int\frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\int \frac{}{\sqrt{}}}{b}\right.}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3121

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\sin}}{b\sqrt{e\sin(c}}\right.}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3042

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\sin}}{b\sqrt{e\sin(c}}\right.}{}\right.}{7b^2}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3120

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\mathrm{si}}}{bd\sqrt{}}\right.}{}\right.}{7}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 3181

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\sin}}{bd\sqrt{}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$\downarrow$ 266

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\sin}}{bd\sqrt{}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$\Big\downarrow \; 756$$

$$e^2 \left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d} - \frac{e^2 \left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2 \left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\text{si}\ldots}}{bd\sqrt{\ldots}} \ldots \right.}{\ldots} \right.}{\ldots} \right.$$

$$\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

$$\Big\downarrow \; 218$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\sin\cdots}}{bd\sqrt{\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$\downarrow 221$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$e^2 \left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\text{si}}}{bd\sqrt{}} \right.$$

$$e^2 \left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d} - \right.$$

$$e^2 \left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d} - \right.$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$\downarrow$ 3042

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$e^2 \left(\frac{2e(e\sin(c+dx))^{5/2}(7(a^2-b^2)-5ab\cos(c+dx))}{35b^2d} - \frac{e^2 \left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2 \left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\mathrm{si}}}{bd\sqrt{}} \right.}{} \right.}{} \right.$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$\downarrow$ 3286

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\mathrm{si}}}{bd\sqrt{}}\right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\right.$$

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\right.$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$\downarrow 3042$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\mathrm{si}}}{bd\sqrt{}}\right.}{}\right.}{}\right.$$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

e^2 $\frac{2a\left(21a^4-49a^2b^2+33b^4\right)\sqrt{\mathrm{si}}}{bd\sqrt{}}$

e^2 $\frac{2e\sqrt{e\sin(c+dx)}\left(21\left(a^2-b^2\right)^2-ab\left(7a^2-12b^2\right)\cos(c+dx)\right)}{3b^2d}-$

e^2 $\frac{2e(e\sin(c+dx))^{5/2}\left(7\left(a^2-b^2\right)-5ab\cos(c+dx)\right)}{35b^2d}-$

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

input
```
Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*e*(e*Sin[c + d*x])^(9/2))/(9*b*d) + (e^2*((2*e*(7*(a^2 - b^2) - 5*a*b*
Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^2*d) - (e^2*((2*e*(21*(a^2 - b
^2)^2 - a*b*(7*a^2 - 12*b^2)*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2*d)
 - (e^2*((2*a*(21*a^4 - 49*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - (21*(a^2 - b^2)^3*((-
2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqr
t[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/
(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*Ellipt
icPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*
EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])
))/b))/(3*b^2)))/(7*b^2)))/b
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.58. $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.58.4 Maple [A] (warning: unable to verify)

Time = 6.22 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.71

method	result	size
default	Expression too large to display	930

input
```
int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(1/45/b^6*(e*sin(d*x+c))^(1/2)*e^4*(5*b^4*cos(d*x+c)^4+9*a^2*b^2*c
os(d*x+c)^2-19*b^4*cos(d*x+c)^2+45*a^4-99*a^2*b^2+59*b^4)-1/8*e^6*(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/b^6*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1
/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/
2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*s
in(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
*e^6*a*(-1/21/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-6*b^4*cos(d*x+c)^4*s
in(d*x+c)+21*a^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-49*a^2*b^2*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/
2),1/2*2^(1/2))+33*b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-14*a^2*b^2*cos(d*x+c
)^2*sin(d*x+c)+30*b^4*cos(d*x+c)^2*sin(d*x+c))+(-a^6+3*a^4*b^2-3*a^2*b^4+b
^6)/b^6*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/
b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1
/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Ellip...
```

3.58.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a), x)
```

3.58.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin(c+d\,x))^{11/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)), x)
```

3.59 $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

3.59.1 Optimal result

Integrand size = 25, antiderivative size = 461

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx &= -\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d}\\
&+\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d}\\
&+\frac{a(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{a(a^2-b^2)^2\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{2a(5a^2-8b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}}\\
&+\frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}\end{aligned}$$

output
```
-(-a^2+b^2)^(7/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(9/2)/d+(-a^2+b^2)^(7/4)*e^(9/2)*arctanh(b^(1/2)*(e*sin(d*
x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+2/15*e^3*(5*a^2-5*b^2-3*a*
b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d-2/7*e*(e*sin(d*x+c))^(7/2)/b/d-a*
(a^2-b^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))
*sin(d*x+c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-a*(a^2-b
^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/5*a*(5*a^2-8
*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^4/d/sin(
d*x+c)^(1/2)
```

3.59.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 36.31 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.81

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{(e\sin(c+dx))^{9/2}\left(\frac{(5a^3-8ab^2)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{\right.\right.\right.}{}}{}\right.}{}$$

$$+\frac{\csc^4(c+dx)(e\sin(c+dx))^{9/2}\left(-\frac{(-28a^2+37b^2)\sin(c+dx)}{42b^3}-\frac{a\sin(2(c+dx))}{5b^2}+\frac{\sin(3(c+dx))}{14b}\right)}{d}$$

input
```
Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

output
```
-1/5*((e*Sin[c + d*x])^(9/2)*(((5*a^3 - 8*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]
*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a
^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2
- b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*Appel
lF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*
Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 +
b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^2*b - 5*b^3)*Cos
[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])
/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(
-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sq
rt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2,
(b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a
 + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x
]^2])))/(b^3*d*Sin[c + d*x]^(9/2)) + (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2
)*(-1/42*((-28*a^2 + 37*b^2)*Sin[c + d*x])/b^3 - (a*Sin[2*(c + d*x)])/(5*b
^2) + Sin[3*(c + d*x)]/(14*b)))/d
```

3.59.3 Rubi [A] (warning: unable to verify)

Time = 2.21 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3174, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 25$$

$$\frac{e^2\int \frac{(b+a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 3344$$

$$\frac{e^2\left(\frac{2e^2\int -\frac{\left(b\left(2a^2-5b^2\right)+a\left(5a^2-8b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{5b^2}+\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int \frac{\left(b\left(2a^2-5b^2\right)+a\left(5a^2-8b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(2a^2-5b^2\right)+a\left(5a^2-8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 3346$$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(5a^2-8b^2\right)\int \sqrt{e\sin(c+dx)}dx}{b}-\frac{5\left(a^2-b^2\right)^2\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{a(5a^2-8b^2)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5(a^2-b^2)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3121

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{a(5a^2-8b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5(a^2-b^2)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{a(5a^2-8b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5(a^2-b^2)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\Big\downarrow$ 3119

3.59. $\int\frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a(5a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5(a^2-b^2)^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{b}-$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 3180

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a(5a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5(a^2-b^2)^2\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+(a^2-}}{d}\right.}{}\right.}{}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 266

$$\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a(5a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5(a^2-b^2)^2\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+(a^2-}}{d}\right.}{}\right.}{}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

3.59. $\int\frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$\downarrow$ 827

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2}}}{2b}\right.}{}\right.}{}\right)}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 218

3.59. $\int\frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx\cdots}{\sqrt[4]{b^2-a^2\cdots}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a\cdots}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b\cdots}\cdots\right)$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 221

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2b}\right.}{}\right.}{}\right.$$

$$\overline{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx} b}$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 3042

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$5\left(a^2-b^2\right)^2 \left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2b} \right.$$

$$e^2 \left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \right.$$

$$e^2 \left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d} - \right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 3286

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{\quad}{\sqrt{\sin(c+dx)}\cdots}}{2b\sqrt{e\sin\cdots}}\cdots\right)}{\cdots}\right)}{\cdots}\right)$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\downarrow$ 3042

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}}}{2b\sqrt{e\sin}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

$\big\downarrow$ 3284

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(a^2-b^2\right)-3ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(5a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(a^2-b^2\right)^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx}{\sqrt[4]{b^2-a^2}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a}}\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

input
```
Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

output
```
(-2*e*(e*Sin[c + d*x])^(7/2))/(7*b*d) + (e^2*((2*e*(5*(a^2 - b^2) - 3*a*b*
Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^2*d) - (e^2*((2*a*(5*a^2 - 8*b
^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c
 + d*x]]) - (5*(a^2 - b^2)^2*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x
])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(S
qrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(
1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x
]])))/b))/(5*b^2)))/b
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.59. $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.59.4 Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.85

method	result	size
default	Expression too large to display	851

input
```
int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(-1/21/b^4*(e*sin(d*x+c))^(3/2)*e^2*(3*b^2*cos(d*x+c)^2+7*a^2-10*b
^2)+1/8*e^4*(a^4-2*a^2*b^2+b^4)/b^6/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln(
(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*
(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)
^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*a*(
1/5/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(10*(1-sin(d*x+c))^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2
))*a^2-16*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-5*(1-sin(d*x+c))^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))*a^2+8*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2+2*b^2*cos(d*x+c)^4-2*b^2*cos(
d*x+c)^2)+(a^4-2*a^2*b^2+b^4)/b^4*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+
b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((
1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/...
```

3.59.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.59.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a), x)
```

3.59.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = \int \frac{\left(e\ \sin\left(c+d\,x\right)\right)^{9/2}}{a+b\ \cos\left(c+d\,x\right)}\,dx$$

input
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)), x)
```

3.60 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

3.60.1 Optimal result

Integrand size = 25, antiderivative size = 474

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx &= \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d}\\
&+\frac{(-a^2+b^2)^{5/4}\,e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d}\\
&-\frac{2a(3a^2-4b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}}\\
&+\frac{a(a^2-b^2)^2\,e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{a(a^2-b^2)^2\,e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}\end{aligned}$$

output
```
(-a^2+b^2)^(5/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(7/2)/d+(-a^2+b^2)^(5/4)*e^(7/2)*arctanh(b^(1/2)*(e*sin(d*x
+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d-2/5*e*(e*sin(d*x+c))^(5/2)/
b/d+2/3*a*(3*a^2-4*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1
/2)/b^4/d/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b
^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2
+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+2/3*e^3*(3*a^2-3*b^2-a*b*cos(d*x+c))*(e
*sin(d*x+c))^(1/2)/b^3/d
```

3.60.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.54 (sec) , antiderivative size = 1955, normalized size of antiderivative = 4.12

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(((-2*a*Cos[c + d*x])/(3*b^2) + Cos[2*(c + d*x)]/(5*b))*Csc[c + d*x]^3*(e*
Sin[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((28*a*b*Cos[c + d*x]^2*(
a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[S
in[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/
(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c +
 d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1
[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (
a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)
/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a +
 b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-10*a^2 + 27*b^2)*Cos[c + d*x
]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*
x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si...
```

3.60.3 Rubi [A] (warning: unable to verify)

Time = 2.22 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.01, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3174, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 25$$

$$\frac{e^2 \int \frac{(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{b} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2 \int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3344$$

$$\frac{e^2\left(\frac{2e^2\int -\frac{b\left(2a^2-3b^2\right)+a\left(3a^2-4b^2\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}\right)}{b} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int \frac{b\left(2a^2-3b^2\right)+a\left(3a^2-4b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}\right)}{b} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int \frac{b\left(2a^2-3b^2\right)-a\left(3a^2-4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3b^2}\right)}{b} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3346$$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{3b^2}\right)}{b} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 3042$$

3.60. $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3121

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3042

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3120

$$\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(\right.\right.}}{b}\right)}{3b^2}\right)}{b}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

3.60. $\int\frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$\downarrow$ 3181

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{be\int\frac{}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(\cdots\right.}}{}\cdots\right.}{}\cdots\right)}{b}\cdots\right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 266

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2\cdots\right.}}{d}\cdots\right.}{}\cdots\right)}{b}\cdots\right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 756

3.60. $\int\frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin\cdots}}{2e\sqrt{\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{ 218}$$

3.60. $\int\frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin\cdots}}{2e\sqrt{\cdots}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \; 221$$

3.60. $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}}\right.}{}\right.}{}\right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3042

3.60. $\int\frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}}\right.}{}\right.}{}\right.$$

$$b$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3286

3.60. $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}}}{2\sqrt{b^2-a^2}\sqrt{}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3042

3.60. $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(a^2-b^2\right)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}}}{2\sqrt{b^2-a^2}\sqrt{}}\right.}{}\right.}{}\right.$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

$\downarrow$ 3284

3.60. $\int\frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3(a^2-b^2)-ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(3a^2-4b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(a^2-b^2\right)^2\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c-}}{\sqrt[4]{b^2-}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)}\right.}{}\right.}{}\right)}{}\right.}{}$$

$$\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

3.60. $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

output
```
(-2*e*(e*Sin[c + d*x])^(5/2))/(5*b*d) + (e^2*((2*e*(3*(a^2 - b^2) - a*b*Co
s[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2*d) - (e^2*((2*a*(3*a^2 - 4*b^2)*E
llipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*
x]]) - (3*(a^2 - b^2)^2*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x
])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqr
t[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/
4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*
Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^
2])*d*Sqrt[e*Sin[c + d*x]])))/b))/(3*b^2)))/b
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.60. $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.60.4 Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.63

method	result
default	$-2eb\left(-\frac{\sqrt{e\sin(dx+c)}\, e^2\left(b^2\left(\cos^2(dx+c)\right)+5a^2-6b^2\right)}{5b^4}+\frac{e^4\left(a^4-2a^2b^2+b^4\right)\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx\cdots}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx\cdots}}\cdots\right.\right.}{8b^4\left(\cdots\right.}\cdots\right.$

input
```
int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(-1/5/b^4*(e*sin(d*x+c))^(1/2)*e^2*(b^2*cos(d*x+c)^2+5*a^2-6*b^2)+
1/8*e^4*(a^4-2*a^2*b^2+b^4)/b^4*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2
)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*
(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*e^4*a*(1/3/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(3*(1-sin(d*x+c))^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/
2),1/2*2^(1/2))*a^2-4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^
2*sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/b^4*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2
)/b),1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.60.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.60.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a), x)
```

3.60.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\ \sin(c+d\,x))^{7/2}}{a+b\ \cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)), x)
```

3.61 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

3.61.1 Optimal result

Integrand size = 25, antiderivative size = 399

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx &= -\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}\\
&+\frac{(-a^2+b^2)^{3/4}\,e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}\\
&-\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{a(a^2-b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{2ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}\end{aligned}$$

output
```
-(-a^2+b^2)^(3/4)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(5/2)/d+(-a^2+b^2)^(3/4)*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*
x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/d-2/3*e*(e*sin(d*x+c))^(3/2)
/b/d+a*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(
1/2))*sin(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+a*(
a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*si
n(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*a*e^2*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/sin(d*x+c)^(1/2)
```

3.61.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.73

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \frac{(e\sin(c+dx))^{5/2}\left(-2\csc(c+dx)+\frac{\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(-\frac{a\sec(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]
```

output
```
((e*Sin[c + d*x])^(5/2)*(-2*Csc[c + d*x] + (Cos[c + d*x]*(a + b*Sqrt[Cos[c
 + d*x]^2])*(-((a*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^
2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]
] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(a^2 - b^2))
 + ((3 + 3*I)*b^2*(-a^2 + b^2)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c +
 d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d
*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2]
+ (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]
]) - 8*a*b^(5/2)*(-a^2 + b^2)^(1/4)*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x
]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/((-a^2 + b^2)^
(5/4)*Sqrt[Cos[c + d*x]^2])))/(4*b^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x]
^(5/2))))/(3*b*d)
```

3.61.3 Rubi [A] (warning: unable to verify)

Time = 1.77 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3174, 25, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\int\frac{(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3042

$$\frac{e^2\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3346

$$\frac{e^2\left(\frac{a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{(a^2-b^2)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3042

$$\frac{e^2\left(\frac{a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3121

$$\frac{e^2\left(\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3042

$$\frac{e^2\left(\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3119

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\big\downarrow$ 3180

3.61. $\int\frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx$

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 266$$

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 827$$

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \; 218$$

3.61. $\int\frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(c+}}}{}\right)}{b}\right)}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 221

$$\frac{e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots 2\right)}{b}\right)}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 3042

3.61. $\int\frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\right.}{b}\right.$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3286

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+d\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3042

3.61. $\int\frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+d\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

$\downarrow$ 3284

$$e^2\left(\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\right.}{bd\left(b-\sqrt{b^2-a\right.}}\right.}{b}\right.}{b}$$

$$\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]
```

3.61. $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

output
```
(-2*e*(e*Sin[c + d*x])^(3/2))/(3*b*d) + (e^2*((2*a*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]]) - ((a^2 - b^2)*((
-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/
2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-
a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*Ellipt
icPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2
*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*
(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b))/b
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.61. $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

3.61.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.60

method	result
default	$-2eb\left(\frac{(e\sin(dx+c))^{\frac{3}{2}}}{3b^2}-\frac{e^2(a^2-b^2)\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}+1\right.\right.}{8b^4\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}\right.$

input
```
int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(1/3*(e*sin(d*x+c))^(3/2)/b^2-1/8*e^2*(a^2-b^2)/b^4/(e^2*(a^2-b^2)
/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x
+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arc
tan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*e^3*a*(-1/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((1-sin(d*
x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))-(a^2
-b^2)/b^2*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1
+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.61.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c)),x)
```

output
```
Timed out
```

3.61.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a), x)
```

3.61.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\,\sin(c+d\,x))^{5/2}}{a+b\,\cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)), x)
```

3.62 $\int \frac{(e \sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

3.62.1 Optimal result

Integrand size = 25, antiderivative size = 410

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx &= \frac{\sqrt[4]{-a^2+b^2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} \\ &+ \frac{\sqrt[4]{-a^2+b^2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} \\ &+ \frac{2ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} \\ &- \frac{a(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{a(a^2-b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{2e\sqrt{e\sin(c+dx)}}{bd}\end{aligned}$$

output
```
(-a^2+b^2)^(1/4)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(3/2)/d+(-a^2+b^2)^(1/4)*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x
+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/d-2*a*e^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*
d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)*e^2*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/b^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)*e^2*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/b^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-2*e*(e*sin(d*x+c
))^(1/2)/b/d
```

3.62.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.11 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.06

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx =$$

$$-\frac{\left(\frac{1}{20}-\frac{i}{20}\right)\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)(e\sin(c+dx))^{3/2}\left(-5(a^2-b^2)\left(2\sqrt[4]{-a^2+b^2}\arctan\left(1-\right.\right.\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

output
```
((-1/20 + I/20)*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*(e*Sin[c + d*x])
^(3/2)*(-5*(a^2 - b^2)*(2*(-a^2 + b^2)^(1/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*(-a^2 + b^2)^(1/4)*ArcTan[1 + (
(1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + (-a^2 + b^2)^(1/
4)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + I*b*Sin[c + d*x]] - (-a^2 + b^2)^(1/4)*Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + (4
 + 4*I)*Sqrt[b]*Sqrt[Sin[c + d*x]]) + (4 + 4*I)*a*b^(3/2)*AppellF1[5/4, 1/
2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]
^(5/2)))/(b^(3/2)*(-a^2 + b^2)*d*Sqrt[Cos[c + d*x]^2]*(a + b*Cos[c + d*x])
*Sin[c + d*x]^(3/2))
```

3.62.3 Rubi [A] (warning: unable to verify)

Time = 1.83 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3174, 25, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$-\frac{e^2\int -\frac{b+a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\int \frac{b+a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{(a^2-b^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{(a^2-b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{e^2\left(\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 3042

$$\frac{e^2\left(\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 3120

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{b}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 3181

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+(a^2-b^2)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-\right.}}}{2\sqrt{b^2-}}\right.}{b}}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 266

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+(a^2-b^2)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+d\right.}}{2\sqrt{b^2-a^2}}\right.}{b}}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 756

3.62. $\int\frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c}{2e\sqrt{b^2-a^2}}\right.}{d}\right.}{b}\right.$$

$$\frac{}{\quad b \quad}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 218

$$e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int\frac{}{\sqrt{}}}{}\right.}{b}\right.$$

$$\frac{}{\quad b \quad}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 221

3.62. $\int\frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\big\downarrow$ 3042

$$\frac{e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\big\downarrow$ 3286

3.62. $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

$$e^2 \left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{(a^2-b^2)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}} - \frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\sin(\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c}}\right.}{b}\right.$$

$$\frac{}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 3042

$$e^2 \left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{(a^2-b^2)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}} - \frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\sin(\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c}}\right.}{b}\right.$$

$$\frac{}{b}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

$\downarrow$ 3284

3.62. $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

$$e^2\left(\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(a^2-b^2)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{Ellip}}{d\sqrt{b^2-a^2}}\right.}{b}\right.$$

$$\frac{b}{}$$

$$\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*e*Sqrt[e*Sin[c + d*x]])/(b*d) + (e^2*((2*a*EllipticF[(c - Pi/2 + d*x)/
2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - ((a^2 - b^2)*((-2*b
*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b
]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a
^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*EllipticP
i[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*Ell
ipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/
b))/b
```

3.62. $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.62. $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx$

3.62.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.60

method	result
default	$-2eb\left(\frac{\sqrt{e\sin(dx+c)}}{b^2}-\frac{e^2\left(a^2-b^2\right)\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{e^2(}{}\right.}\right.\right.}{8b^2\left(a^2e^2-b^2e^2\right)}\right.$

input
```
int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*((e*sin(d*x+c))^(1/2)/b^2-1/8*e^2*(a^2-b^2)/b^2*(e^2*(a^2-b^2)/b^2
)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-
(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)
^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1
)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*a*(-1/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Elliptic
F((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+(-a^2+b^2)/b^2*(-1/2/(-a^2+b^2)^(1/2)/
b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1
/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*
x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.62.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.62.6 Sympy [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin{(c+dx)})^{\frac{3}{2}}}{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral((e*sin(c + d*x))**(3/2)/(a + b*cos(c + d*x)), x)
```

3.62.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin{(dx+c)})^{\frac{3}{2}}}{b\cos{(dx+c)}+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a), x)
```

3.62.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = \int \frac{(e\ \sin(c+d\,x))^{3/2}}{a+b\ \cos(c+d\,x)}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)), x)
```

3.63 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$

3.63.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d}+\frac{\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}d}+\frac{ae\,\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{ae\,\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$

output
```
-arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a
^2+b^2)^(1/4)/d/b^(1/2)+arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(1/4)/d/b^(1/2)-a*e*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*
x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/d/(b-(-a^2+b^2)^(1
/2))/(e*sin(d*x+c))^(1/2)-a*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)
```

3.63.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.65 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\sqrt{e\sin(c+dx)}\left(\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2\arctan\left(1-\frac{(1+i)\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\arctan\left(1+\frac{(1+i)\sqrt{b}\sqrt{\sin}}{\sqrt[4]{-a^2}}\right.\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output
```
(2*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(((1/8 +
 I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4
)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)
^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^
2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2))))/(d*Sqrt[Cos[c + d*x
]^2]*(a + b*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

3.63.3 Rubi [A] (warning: unable to verify)

Time = 1.14 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3180$$

$$-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+(a^2-b^2)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \; 266$$

$$-\frac{2be\int \frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+(a^2-b^2)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \; 827$$

$$-\frac{2be\left(\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \; 218$$

$$-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}$$

$$\Big\downarrow \; 221$$

$$-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \; 3042$$

3.63. $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$

$$-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \text{ 3286}$$

$$-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}$$

$$\Big\downarrow \text{ 3284}$$

3.63. $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx$

$$-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3
/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(
-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*Ellip
ticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(
2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b
*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])
```

3.63.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.63. $\int \frac{\sqrt{e \sin(c+dx)}}{a+b\cos(c+dx)}\,dx$

3.63.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.64

method	result
default	$-\frac{e\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}-1\right.\right.}{4b\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}$

input
```
int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*e/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*s
in(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2
-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2
)-1))+1/2*a*e*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/b*(EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*
(-a^2+b^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),
1/2*2^(1/2))*b-EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/
2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2
)^(1/2))*b,1/2*2^(1/2))*b)/(-b+(-a^2+b^2)^(1/2))/(b+(-a^2+b^2)^(1/2))/cos(
d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.63.5 Fricas [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.63.6 Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin{(c+dx)}}}{a+b\cos{(c+dx)}}\,dx$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c)),x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x)), x)
```

3.63.7 Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin{(dx+c)}}}{b\cos{(dx+c)}+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.63.8 Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\sin{(dx+c)}}}{b\cos{(dx+c)}+a}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sqrt{e\ \sin\left(c+d\,x\right)}}{a+b\ \cos\left(c+d\,x\right)}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)
```

output
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)
```

3.64 $\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

3.64.1 Optimal result

Integrand size = 25, antiderivative size = 307

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx &= \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}}\\ &+\frac{a\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\ &+\frac{a\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\end{aligned}$$

output
```
arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^
2+b^2)^(3/4)/d/e^(1/2)+arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(3/4)/d/e^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/d/(a^2-b*(b-(-a^2+b^2)^
(1/2)))/(e*sin(d*x+c))^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^
(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x
+c))^(1/2)
```

3.64.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.62 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{10(a+b)\,\mathrm{AppellF1}}{de(a+b\cos(c+dx))\left(5(a+b)\,\mathrm{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+2\left(-2(a\right.\right.}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(10*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan
[(c + d*x)/2]^2)/(a + b)]*Sqrt[e*Sin[c + d*x]])/(d*e*(a + b*Cos[c + d*x])*
(5*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[
(c + d*x)/2]^2)/(a + b)] + 2*(-2*(a - b)*AppellF1[5/4, -1/2, 2, 9/4, -Tan[
(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[
5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a +
b)])*Tan[(c + d*x)/2]^2))
```

3.64.3 Rubi [A] (warning: unable to verify)

Time = 1.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 3181$$

$$-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+(a^2-b^2)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$\downarrow$ 266

$$-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+(a^2-b^2)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$\downarrow$ 756

$$-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$\downarrow$ 218

$$-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}$$

$\downarrow$ 221

$$-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}$$

$\downarrow$ 3042

3.64. $\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx$

$$-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}$$

$$\Big\downarrow \text{ 3286}$$

$$-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}$$

$$\Big\downarrow \text{ 3284}$$

3.64. $\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx$

$$-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

input
```
Int[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
(-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(S
qrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x]
)/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*Elli
pticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (
a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]
])
```

3.64.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

3.64.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.69

method	result
default	$-\frac{b e \left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.}{4\left(a^2e^2-b^2e^2\right)}$

input
```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*b*e*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d
*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)-1))+1/2*a*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*(EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2))
,1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^
2+b^2)^(1/2)),1/2*2^(1/2))*b+EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^
2)^(1/2))*b,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((1-sin(d*x+c))^(1/2),
1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b)/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(
1/2))/(b+(-a^2+b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.64.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.64.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))), x)
```

3.64.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.64.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin(c+d\,x)}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

3.65 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

3.65.1 Optimal result

Integrand size = 25, antiderivative size = 426

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx &= -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}} \\ &+ \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}} + \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} \\ &- \frac{ab\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\ &- \frac{ab\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\ &- \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}}\end{aligned}$$

output
```
-b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a
^2+b^2)^(5/4)/d/e^(3/2)+b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2
+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^
2)/d/e/(e*sin(d*x+c))^(1/2)+a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)
^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b-(-a^2+b^2)^(1/2))/(e*si
n(d*x+c))^(1/2)+a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)+2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/
e^2/sin(d*x+c)^(1/2)
```

3.65.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.03 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(-b+a\cos(c+dx))\sin(c+dx)}{(a^2-b^2)\,d(e\sin(c+dx))^{3/2}}$$

$$-\frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{a\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{\cdots}\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)*d*(e*Sin[c + d*x])^(3
/2)) - (Sin[c + d*x]^(3/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/
4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1,
7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)
)*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c
+ d*x])*(1 - Sin[c + d*x]^2)) + (2*(a^2 + b^2)*Cos[c + d*x]*(((1/8 + I/8)*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*
x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)*(a + b)
*d*(e*Sin[c + d*x])^(3/2))
```

3.65.3 Rubi [A] (warning: unable to verify)

Time = 1.92 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3175, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3175}$$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{\left(a^2+b\cos(c+dx)a+b^2\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\left(a^2+b\cos(c+dx)a+b^2\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b\sin\left(c+dx+\frac{\pi}{2}\right)a+b^2\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3346

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx+a\int \sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+a\int \sqrt{e\sin(c+dx)}dx}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3121

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3119

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3180

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}\right.}{e^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 266

3.65. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}\cdots}{2b}\cdots\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 827

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\cdots\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 218

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\cdots}{\cdots}\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 221

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{\cdots}}\right)}{d}\right.}{e^2\left(a^2-b^2\right)}$$

$\downarrow$ 3042

3.65. $\int\frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$b^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{}}{}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{}}\right)}{d}\right.$$

$$\overline{\qquad e^2\left(a^2-b^2\right)\qquad}$$

$\downarrow$ 3286

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$b^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c-}{\sqrt[4]{b^2-}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.$$

$$\overline{\qquad e^2\left(a^2-b^2\right)\qquad}$$

$\downarrow$ 3042

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-$$

$$b^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c-}{\sqrt[4]{b^2-}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.$$

$$\overline{\qquad e^2\left(a^2-b^2\right)\qquad}$$

$\downarrow$ 3284

3.65. $\int\frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{2(b-a\cos(c+dx))}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+\cdots}}{\cdots}\right)}{e^2\left(a^2-b^2\right)}$$

input
```
Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - ((2*a*El
lipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]
) + b^2*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)
]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c
+ d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (
a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Ell
ipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 - b^2)*e^
2)
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

3.65. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.65.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.82

method	result
default	$-be\left(-\frac{2}{e^{2}\left(a^{2}-b^{2}\right)\sqrt{e\sin\left(dx+c\right)}}-\frac{\sqrt{2}\left(\ln\left(\frac{e\sin\left(dx+c\right)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin\left(dx+c\right)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e\sin\left(dx+c\right)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin\left(dx+c\right)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin\left(dx+c\right)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}+1\right)\right.}{4e^{2}\left(a-b\right)\left(a+b\right)\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}\right.$

input
```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*e*(-2/e^2/(a^2-b^2)/(e*sin(d*x+c))^(1/2)-1/4/e^2/(a-b)/(a+b)/(e^2*(a^2
-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*si
n(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+
2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arcta
n(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/2*(4*(1-si
n(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d
*x+c))^(1/2),1/2*2^(1/2))*a^2-2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*
x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(1-si
n(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(
d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2-(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2)
,1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/
2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^2-4*a^2*cos(d*x+c)^2)*a/e/(b+(-
a^2+b^2)^(1/2))/(-b+(-a^2+b^2)^(1/2))/(a+b)/(a-b)/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2))/d
```

3.65.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.65.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}\,(a+b\cos{(c+dx)})}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))), x)
```

3.65.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

3.65.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

3.66 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

3.66.1 Optimal result

Integrand size = 25, antiderivative size = 447

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx &= \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}\,de^{5/2}} \\ &+ \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}\,de^{5/2}} + \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} \\ &+ \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3(a^2-b^2)\,de^2\sqrt{e\sin(c+dx)}} \\ &- \frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}} \\ &- \frac{ab^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}\end{aligned}$$

output
```
b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(7/4)/d/e^(5/2)+b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-b
^2)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*
x+c)^(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)+a*b^2*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a
^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*b^2*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)
```

3.66.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 1192, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(-b+a\cos(c+dx))\sin(c+dx)}{3\left(a^2-b^2\right)d(e\sin(c+dx))^{5/2}}$$

$$+\frac{\sin^{\frac{5}{2}}(c+dx)\left(\frac{2ab\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\cdots\right.\right.}{4\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
(-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/(3*(a^2 - b^2)*d*(e*Sin[c + d*x])^
(5/2)) + (Sin[c + d*x]^(5/2)*((2*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c
+ d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 -
b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2
)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a
^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c
 + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (
b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5
/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c
 + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (2*(a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c +
d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c
+ d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c +
d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2]
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + ...
```

3.66.3 Rubi [A] (warning: unable to verify)

Time = 1.95 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.96, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3175, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3175}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{2\int -\frac{a^2+b\cos(c+dx)a-3b^2}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a^2+b\cos(c+dx)a-3b^2}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{a^2-b\sin\left(c+dx-\frac{\pi}{2}\right)a-3b^2}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3346$$

$$\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-3b^2\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3121$$

$$\frac{\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3120$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3181$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-\ldots\right.}}{2\sqrt{b^2-a^2}\ldots}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

3.66. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \ 266$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}d}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 756$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int\frac{}{\sqrt{e\sin(c}}}{}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \ 221$$

3.66. $\int\frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\right.$$

$$\overline{\qquad 3e^2\left(a^2-b^2\right)\qquad}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\right.$$

$$\overline{\qquad 3e^2\left(a^2-b^2\right)\qquad}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3286$$

$$\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+}}\right.$$

$$\overline{\qquad 3e^2\left(a^2-b^2\right)\qquad}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

3.66. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+\right.}}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

$\downarrow$ 3284

$$\frac{\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-3b^2\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{Elliptic}}{d\sqrt{b^2-a^2}\left(b-\right.}\right.}{3e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}$$

input

```
Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

output

```
(2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + ((2*
a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d
*x]]) - 3*b^2*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 +
 b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[
e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)
)))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin
[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Sin[c + d*x]])))/(3*(a^2 - b^2)*e^2)
```

3.66. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.66. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

3.66.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.59

method	result
default	$-2eb\left(-\frac{b^{2}\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e\sin(dx+c)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arc}\ldots\right.}{8e^{2}(a-b)(a+b)\left(a^{2}e^{2}-b^{2}e^{2}\right)}\right.$

input
```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(-1/8/e^2/(a-b)/(a+b)*b^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e
^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/
2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1
/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))-1/3/e^2/(a^2-b^2)/(e*sin(d
*x+c))^(3/2))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a/e^2*(1/3/(a^2-b^2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((1-sin(d*x+c))^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))+2*cos(d*x+c)^2*sin(d*x+c))-1/(a-b)/(a+b)*b^2*(-1/2/(-a^2+b^2)^(1/2)/b*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2
),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

3.66.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\,(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(e*sin(d*x + c))/((b*e^3*cos(d*x + c)^3 + a*e^3*cos(d*x + c)
^2 - b*e^3*cos(d*x + c) - a*e^3)*sin(d*x + c)), x)
```

3.66.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\sin{(c+dx)})^{\frac{5}{2}}\,(a+b\cos{(c+dx)})}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

output
```
Integral(1/((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x))), x)
```

3.66.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}+a)\,(e\sin{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

3.66.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))), x)
```

3.67 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

3.67.1 Optimal result

Integrand size = 25, antiderivative size = 501

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx =
&-\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}\,de^{7/2}}+\frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}\,de^{7/2}} \\
&+\frac{2(b-a\cos(c+dx))}{5\,(a^2-b^2)\,de(e\sin(c+dx))^{5/2}}-\frac{2(5b^3+a(3a^2-8b^2)\cos(c+dx))}{5\,(a^2-b^2)^2\,de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{ab^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&-\frac{2a(3a^2-8b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5\,(a^2-b^2)^2\,de^4\sqrt{\sin(c+dx)}}
\end{aligned}$$

output
```
-b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a
^2+b^2)^(9/4)/d/e^(7/2)+b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2
+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-
b^2)/d/e/(e*sin(d*x+c))^(5/2)-2/5*(5*b^3+a*(3*a^2-8*b^2)*cos(d*x+c))/(a^2-
b^2)^2/d/e^3/(e*sin(d*x+c))^(1/2)-a*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-
a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b-(-a^2+b^2)^
(1/2))/(e*sin(d*x+c))^(1/2)-a*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(
1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^
2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b+(-a^2+b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+2/5*a*(3*a^2-8*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))
*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^4/sin(d*x+c)^(1/2)
```

3.67.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.66 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{\sin^{\frac{7}{2}}(c+dx)\left(\frac{4a^2b-14b^3+(-7a^3+12ab^2)\cos(c+dx)+10b^3\cos(2(c+dx))+3a^3}{2(a^2-b^2)^2\sin^{\frac{5}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(Sin[c + d*x]^(7/2)*((4*a^2*b - 14*b^3 + (-7*a^3 + 12*a*b^2)*Cos[c + d*x]
+ 10*b^3*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)] - 8*a*b^2*Cos[3*(c + d*
x)])/(2*(a^2 - b^2)^2*Sin[c + d*x]^(5/2)) - (Cos[c + d*x]*(a + b*Sqrt[Cos[
c + d*x]^2])*((a*(3*a^2 - 8*b^2)*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/
4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1,
7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)
))/(Sqrt[b]*(-a^2 + b^2)) + (24*(3*a^4 - 8*a^2*b^2 - 5*b^4)*(((1/8 + I/8)*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2))))/Sqrt[Cos[c + d*x]^2]))/(1
2*(a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]))))/(5*d*(e*Sin[c + d*x])^(7/2))
```

3.67.3 Rubi [A] (warning: unable to verify)

Time = 2.49 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.99, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3175, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{2\int -\frac{3a^2+3b\cos(c+dx)a-5b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{3a^2+3b\cos(c+dx)a-5b^2}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{3a^2-3b\sin\left(c+dx-\frac{\pi}{2}\right)a-5b^2}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3345}$$

$$\frac{-\frac{2\int \frac{\left(3a^4-8b^2a^2+b\left(3a^2-8b^2\right)\cos(c+dx)a-5b^4\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{\int \frac{\left(3a^4-8b^2a^2+b\left(3a^2-8b^2\right)\cos(c+dx)a-5b^4\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(3a^4-8b^2a^2+b\left(3a^2-8b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)a-5b^4\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3346}$$

$$\frac{-\frac{a\left(3a^2-8b^2\right)\int \sqrt{e\sin(c+dx)}dx-5b^4\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2(a^2-b^2)}-\frac{2\left(a\left(3a^2-8b^2\right)\cos(c+dx)+5b^3\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \textbf{3042}$$

3.67. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

$$-\frac{-\frac{a(3a^2-8b^2)\int\sqrt{e\sin(c+dx)}dx-5b^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}-\frac{2(a(3a^2-8b^2)\cos(c+dx)+5b^3)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3121}$$

$$-\frac{-\frac{\frac{a\left(3a^2-8b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-5b^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}-\frac{2(a(3a^2-8b^2)\cos(c+dx)+5b^3)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{\frac{a\left(3a^2-8b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-5b^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}-\frac{2(a(3a^2-8b^2)\cos(c+dx)+5b^3)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}-\frac{2(a(3a^2-8b^2)\cos(c+dx)+5b^3)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}}{5e^2(a^2-b^2)}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3180}$$

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\cdots\right)}{e^2(a^2-b^2)}\cdots$$

$$\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 266}$$

3.67. $\int\frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

$$-\frac{\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 827

$$-\frac{\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 218

$$-\frac{\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(c+dx)}}}{}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 221

3.67. $\int\frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be}{}\right.}{e^2(a^2-b^2)}$$

$$5e^2\left(a^2-b^2\right)$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be}{}\right.}{e^2(a^2-b^2)}$$

$$5e^2\left(a^2-b^2\right)$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3286

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{e^2(a^2-b^2)}$$

$$5e^2\left(a^2-b^2\right)$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3042

3.67. $\int\frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

$\downarrow$ 3284

$$-\frac{\frac{2a\left(3a^2-8b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-5b^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{}{b-}\right.}{bd\left(b-\sqrt{b^2-a^2}\right)}\right.}{e^2(a^2-b^2)}}{5e^2\left(a^2-b^2\right)}$$

$$\frac{2(b-a\cos(c+dx))}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}$$

input
```
Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(2*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) + ((-2
*(5*b^3 + a*(3*a^2 - 8*b^2)*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c +
 d*x]]) - ((2*a*(3*a^2 - 8*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Si
n[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) - 5*b^4*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt
[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e
]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)
*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2]
)*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqr
t[e*Sin[c + d*x]])))/((a^2 - b^2)*e^2))/(5*(a^2 - b^2)*e^2)
```

3.67. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.67. $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.67.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.01

method	result	size
default	Expression too large to display	1007

input
```
int(1/(a+cos(d*x+c)*b)/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*e*b*(1/8*b^2/e^4/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln
((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2
*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+
c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2
)^(1/4)*(e*sin(d*x+c))^(1/2)-1))-1/5/e^2/(a+b)/(a-b)/(e*sin(d*x+c))^(5/2)+
1/e^4/(a-b)^2/(a+b)^2*b^2/(e*sin(d*x+c))^(1/2))-1/10/e^3*(12*(1-sin(d*x+c)
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(
1/2),1/2*2^(1/2))*a^4-32*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2-6*(1-sin(d
*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+
c))^(1/2),1/2*2^(1/2))*a^4+16*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2-5*(1-
sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((1-si
n(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3
-5*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi
((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4+5*(1-sin(d
*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3-5*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((...
```

3.67.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.67.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.67.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)\,(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{7/2}\,(a+b\,\cos(c+d\,x))}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))),x)
```

output
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))), x)
```

3.68 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

3.68.1 Optimal result

Integrand size = 25, antiderivative size = 557

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d}\\
&+\frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{11/2}d}\\
&-\frac{3(21a^4-28a^2b^2+5b^4)\,e^6\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}}\\
&+\frac{9a^2(a^2-b^2)^2\,e^6\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{9a^2(a^2-b^2)^2\,e^6\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d}\\
&-\frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d}+\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}\end{aligned}$$

output
```
9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/b^(11/2)/d+9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctanh(b^(
1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d-9/35*e^3*(7
*a-5*b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d+e*(e*sin(d*x+c))^(9/2)/b/d/(
a+b*cos(d*x+c))+3/7*(21*a^4-28*a^2*b^2+5*b^4)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-b^2)^2*e
^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticP
i(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-
b^2)^2*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(
d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+3/7*e
^5*(21*a*(a^2-b^2)-b*(7*a^2-5*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

3.68.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.67 (sec) , antiderivative size = 2029, normalized size of antiderivative = 3.64

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((((-28*a^2 + 17*b^2)*Cos[c + d*x])/(14*b^4) + (-a^2 + b^2)^2/(b^5*(a + b*
Cos[c + d*x])) + (2*a*Cos[2*(c + d*x)])/(5*b^3) - Cos[3*(c + d*x)]/(14*b^2
))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*((
2*(35*a^4 - 126*a^2*b^2 + 75*b^4)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d
*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2
)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(
1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
 + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2
- b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c +
d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2
*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4,
 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c +
d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Si
n[c + d*x]^2)) + (2*(70*a^3*b - 93*a*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin
[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b...
```

3.68.3 Rubi [A] (warning: unable to verify)

Time = 2.79 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3172, 25, 3042, 3344, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{11/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3172}$$

$$\frac{9e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 25$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\int\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{2b}$$

$$\Big\downarrow \ 3042$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\int\frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \ 3344$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e^2\int-\frac{\left(2ab+\left(7a^2-5b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{2(a+b\cos(c+dx))}dx}{7b^2}+\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}\right)}{2b}$$

$$\Big\downarrow \ 27$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\int\frac{\left(2ab+\left(7a^2-5b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{7b^2}\right)}{2b}$$

$$\Big\downarrow \ 3042$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\int\frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(2ab+\left(7a^2-5b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{7b^2}\right)}{2b}$$

$$\Big\downarrow \ 3344$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e^2\int-\frac{2ab\left(7a^2-8b^2\right)+\left(21a^4-28b^2a^2+5b^4\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5\cdots\right.\right.}{3b^2d}\cdots\right.}{7b^2}\cdots\right.}{2b}$$

3.68. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}dx$

$\Big\downarrow$ 27

$$\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2ab\left(7a^2-8b^2\right)+\left(21a^4-28b^2a^2+5b^4\right)\mathrm{c}\cdots}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\cdots}}{3b^2}\cdots\right.}{7b^2}\cdots\right.}{2b}\cdots$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$\Big\downarrow$ 3042

$$\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2ab\left(7a^2-8b^2\right)-\left(21a^4-28b^2a^2+5b^4\right)\mathrm{s}\cdots}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+d\cdots\right.\right.}}{3b^2}\cdots\right.}{7b^2}\cdots\right.}{2b}\cdots$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$\Big\downarrow$ 3346

$$\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}d\cdots}{b}\cdots\right.}{\cdots}\right.}{7b^2}\cdots\right.}{2b}\cdots$$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$\Big\downarrow$ 3042

3.68. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}\cdots\right)}{\cdots}\right)}{7b^2}\cdots\right)}{2b}$$

$\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\int\frac{\cdots}{\sqrt{\sin\cdots}}}{b\sqrt{e\sin(c+dx)}}\cdots\right)}{\cdots}\right)}{7b^2}\cdots\right)}{2b}$$

$\downarrow$ 3042

3.68. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\mathrm{si}}}}{b\sqrt{e\sin(c+dx)}}\right.}{}\right.}{7b^2}\right.}{2b}$$

$\downarrow$ 3120

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-\frac{9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{7b^2}\right.}{2b}$$

$\downarrow$ 3181

3.68. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

$\downarrow$ 756

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

↓ 218

3.68. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}-$$

$$e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.$$

$$e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d}-\right.$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d}-\right.$$

3.68. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$\downarrow$ 221

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\downarrow 3042$$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

3.67. $\downarrow$ 3286

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\mathrm{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

3.68.

$$\downarrow 3284$$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} -$$

$$9e^2\left(\frac{2e(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^2d} - \frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(21a\left(a^2-b^2\right)-b\left(7a^2-5b^2\right)\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2\left(21a^4-28a^2b^2+5b^4\right)\sqrt{\sin(c+dx)}\,\text{Ellip}}{bd\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.$$

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

input
```
Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(e*(e*Sin[c + d*x])^(9/2))/(b*d*(a + b*Cos[c + d*x])) - (9*e^2*((2*e*(7*a
- 5*b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^2*d) - (e^2*((2*e*(21*a*
(a^2 - b^2) - b*(7*a^2 - 5*b^2)*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2
*d) - (e^2*((2*(21*a^4 - 28*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - (21*a*(a^2 - b^2)^2*(
(-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(S
qrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x]
)/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*Elli
pticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (
a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]
])))/b))/(3*b^2)))/(7*b^2)))/(2*b)
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.68. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.68.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1657 vs. 2(579) = 1158.

Time = 18.68 (sec) , antiderivative size = 1658, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1658

input
```
int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(-1/5/b^6*(e*sin(d*x+c))^(1/2)*e^2*(b^2*cos(d*x+c)^2+10*a^2-11
*b^2)+e^4/b^6*((-1/4*a^4+1/2*a^2*b^2-1/4*b^4)*(e*sin(d*x+c))^(1/2)/(-b^2*c
os(d*x+c)^2*e^2+a^2*e^2)+9/32*(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2)/b^2)^(1/4
)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(
a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)
))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*ar
ctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))+(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*e^6*(1/7/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(
-2*b^4*cos(d*x+c)^4*sin(d*x+c)+35*a^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-49*a^
2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipti
cF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+11*b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))
-14*a^2*b^2*cos(d*x+c)^2*sin(d*x+c)+10*b^4*cos(d*x+c)^2*sin(d*x+c))-(-7*a^
6+15*a^4*b^2-9*a^2*b^4+b^6)/b^6*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(...
```

3.68.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.68.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^2, x)
```

3.68.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^2, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\ \sin\left(c+d\,x\right))^{11/2}}{\left(a+b\ \cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2, x)
```

3.69 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

3.69.1 Optimal result

Integrand size = 25, antiderivative size = 473

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} \\
&+\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} \\
&-\frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&-\frac{7a^2(a^2-b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&+\frac{7(5a^2-3b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} \\
&-\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d}+\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}
\end{aligned}
$$

output
```
-7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+
b^2)^(1/4)/e^(1/2))/b^(9/2)/d+7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctanh(b^(1/
2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d-7/15*e^3*(5*a-
3*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d+e*(e*sin(d*x+c))^(7/2)/b/d/(a+b
*cos(d*x+c))+7/2*a^2*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b
^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x
+c))^(1/2)+7/2*a^2*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2
)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c
))^(1/2)-7/5*(5*a^2-3*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)
```

3.69.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.43 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.56

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{(e\sin(c+dx))^{9/2}\left(8b^{3/2}(-35a^2+18b^2-14ab\cos(c+dx)+3b^2\cos(2(c+dx)\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((e*Sin[c + d*x])^(9/2)*(8*b^(3/2)*(-35*a^2 + 18*b^2 - 14*a*b*Cos[c + d*x]
 + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x]^(3/2) + 7*Cos[c + d*x]*(a + b*Sqrt
[Cos[c + d*x]^2])*(-(((5*a^2 - 3*b^2)*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2
)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2
, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^
(3/2)))/(a^2 - b^2)) + (48*a*b^(5/2)*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*
Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + L
og[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/
2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]
^(3/2))/(3*(a^2 - b^2))))/Sqrt[Cos[c + d*x]^2])))/(120*b^(9/2)*d*(a + b*Co
s[c + d*x])*Sin[c + d*x]^(9/2))
```

3.69.3 Rubi [A] (warning: unable to verify)

Time = 2.13 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{7e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\int\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{2b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\int\frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$\Big\downarrow$ 3344

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e^2\int-\frac{\left(2ab+\left(5a^2-3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{5b^2}+\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}\right)}{2b}$$

$\Big\downarrow$ 27

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\int\frac{\left(2ab+\left(5a^2-3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{5b^2}\right)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2ab+\left(5a^2-3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2}\right)}{2b}$$

$\Big\downarrow$ 3346

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{\left(5a^2-3b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5a\left(a^2-b^2\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{5b^2}\right)}{2b}$$

$\Big\downarrow$ 3042

3.69. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{(5a^2-3b^2)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5a(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}}{}$$

$\Big\downarrow$ 3121

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{(5a^2-3b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5a(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}}{}$$

$\Big\downarrow$ 3042

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{(5a^2-3b^2)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5a(a^2-b^2)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}}{}$$

$\Big\downarrow$ 3119

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b}$$

$\downarrow$ 3180

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e}{d}\right.}{}\right.}{}\right.}{2b}$$

$\downarrow$ 266

$$\frac{\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{}}{d}\right.}{}\right.}{}\right.}{2b}$$

$\downarrow$ 827

3.69. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-$$

$$7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{\cdots}}{2b}\right.}{\cdots}\right.}{\cdots}\right)}{\cdots}\right.$$

$\downarrow$ 218

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-$$

$$\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{2b\cdots}$$

$\downarrow$ 221

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-$$

$$7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+\right.}}{2b}\right.}{}\right.}{}\right.$$

$$2b$$

↓ 3042

3.69. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d} - \frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5a\left(a^2-b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+\right.}}{2b}\right.}{}\right.}{}\right.}{2b}$$

↓ 3286

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}-\frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d}-\frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5a\left(a^2-b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}\cdots\right.}}{2b\sqrt{e\sin(c+dx)}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{2b\cdots}$$

$\downarrow$ 3042

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d} - \frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5a\left(a^2-b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{}\right.}{}\right.}{2b}$$

$\downarrow$ 3284

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{7e^2\left(\frac{2e(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^2d} - \frac{e^2\left(\frac{2\left(5a^2-3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5a\left(a^2-b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}} - \frac{\mathrm{ar}\cdots}{\cdots}\cdots\right)}{d}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{2b}$$

input
```
Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

output
```
(e*(e*Sin[c + d*x])^(7/2))/(b*d*(a + b*Cos[c + d*x])) - (7*e^2*((2*e*(5*a
- 3*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^2*d) - (e^2*((2*(5*a^2 -
 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[S
in[c + d*x]]) - (5*a*(a^2 - b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c +
 d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTan
h[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^
2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*
Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]])))/b))/(5*b^2)))/(2*b)
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.69. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.69.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1627 vs. 2(499) = 998.

Time = 17.78 (sec) , antiderivative size = 1628, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	1628

input
```
int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(1/3*(e*sin(d*x+c))^(3/2)/b^4-e^2/b^4*((-1/4*a^2+1/4*b^2)*(e*s
in(d*x+c))^(3/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+1/8*(7/4*a^2-7/4*b^2)/b^2
/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)
+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+
1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))+(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*(-1/5/b^4/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*(30*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*El
lipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-16*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(
1/2))*b^2-15*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+8*(1-sin(d*x+c))^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^
(1/2))*b^2+2*b^2*cos(d*x+c)^4-2*b^2*cos(d*x+c)^2)-(5*a^4-6*a^2*b^2+b^4)/b^
4*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(
d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^...
```

3.69.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin{(dx+c)})^{\frac{9}{2}}}{(b\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^2, x)
```

3.69.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin{(dx+c)})^{\frac{9}{2}}}{(b\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^2, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin(c+d\,x))^{9/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2, x)
```

3.70 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

3.70.1 Optimal result

Integrand size = 25, antiderivative size = 487

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d} \\
&+ \frac{5a\sqrt[4]{-a^2+b^2}e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d} \\
&+ \frac{5(3a^2-b^2)\,e^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2(a^2-b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}
\end{aligned}
$$

output
```
5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(7/2)/d+5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctanh(b^(1/2
)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d+e*(e*sin(d*x+c)
)^(5/2)/b/d/(a+b*cos(d*x+c))-5/3*(3*a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2
^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2)*e^4*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2
)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c
)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-5/3*e^3*(3
*a-b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^3/d
```

3.70.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.65 (sec) , antiderivative size = 1956, normalized size of antiderivative = 4.02

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(((2*Cos[c + d*x])/(3*b^2) + (-a^2 + b^2)/(b^3*(a + b*Cos[c + d*x])))*Csc[
c + d*x]^3*(e*Sin[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((2*(3*a^2
- 5*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1
- (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^
2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]
] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2
 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)
*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x
]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 +
Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*a*b*C
os[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*
Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/...
```

3.70.3 Rubi [A] (warning: unable to verify)

Time = 2.25 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.98, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{5e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\int\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\int\frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \text{3344}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e^2\int-\frac{2ab+\left(3a^2-b^2\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}\right)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\int\frac{2ab+\left(3a^2-b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\int\frac{2ab-\left(3a^2-b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3b^2}\right)}{2b}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{3b^2}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

3.70. $\int\frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}dx$

$$\frac{5e^2\left(\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{2b}$$

$$\Big\downarrow \; 3121$$

$$\frac{5e^2\left(\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{5e^2\left(\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{2b}$$

$$\Big\downarrow \; 3120$$

$$\frac{5e^2\left(\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{3b^2}\right)}{2b}$$

$$\Big\downarrow \; 3181$$

3.70. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+(\cdots\right.}}{d}\cdots\right.}{\cdots}}{\cdots}\right)}{2b\cdots}$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-\frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{\cdots}}{d}\cdots\right.}{\cdots}}{3b\cdots}\right)}{2b}$$

$\downarrow$ 756

3.70. $\int\frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} -$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3a\left(a^2-b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d}{2e\sqrt{b^2-a^2}}\right.}{}\right.}{}\right.}{}\right.$$

$\big\downarrow$ 218

3.70. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d}{2e\sqrt{b^2-a^2}}\right.}{}\right.}{}\right.}{}\right.$$

$$2b$$

$\downarrow$ 221

3.70. $\int\frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+d\right.}}{2\sqrt{b^2-a^2}}\right.}{}\right.}{}\right.$$

$$\overline{\qquad\qquad 2b}$$

$\big\downarrow$ 3042

3.70. $\int\frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} -$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3a\left(a^2-b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+d\right.}}{2\sqrt{b^2-a^2}}\right.}{}\right.}{}\right.$$

$$2b$$

$\downarrow$ 3286

3.70. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3a\left(a^2-b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx}}\right.}{}\right)}{}\right)}{2b}$$

$\downarrow$ 3042

3.70. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}-$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d}-\frac{e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3a\left(a^2-b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx}}\right.}{}}{}\right.$$

$$2b$$

$\downarrow$ 3284

3.70. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} -$$

$$3a\left(a^2-b^2\right)\left(- \frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}} - \cdots\right.}{d}\right.$$

$$e^2\left(\frac{2\left(3a^2-b^2\right)\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \cdots\right.$$

$$5e^2\left(\frac{2e\sqrt{e\sin(c+dx)}(3a-b\cos(c+dx))}{3b^2d} - \cdots\right.$$

$$\cdots\ 2b$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

3.70. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

output
```
(e*(e*Sin[c + d*x])^(5/2))/(b*d*(a + b*Cos[c + d*x])) - (5*e^2*((2*e*(3*a
- b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^2*d) - (e^2*((2*(3*a^2 - b^2)
*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c +
d*x]]) - (3*a*(a^2 - b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d
*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(S
qrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(
3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*
d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 +
b^2])*d*Sqrt[e*Sin[c + d*x]])))/b))/(3*b^2)))/(2*b)
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.70. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. 2(513) = 1026.

Time = 17.46 (sec) , antiderivative size = 1501, normalized size of antiderivative = 3.08

method	result	size
default	Expression too large to display	1501

input
```
int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*((e*sin(d*x+c))^(1/2)/b^4-e^2/b^4*((-1/4*a^2+1/4*b^2)*(e*sin(d
*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+5/32*(a^2-b^2)*(e^2*(a^2-b^2)
/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2
)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x
+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/
b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/
2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))
)+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*(-1/3/b^4/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)*(9*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-4*(1-sin(d*x+c))^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^
(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c))-1/b^4*(5*a^4-6*a^2*b^2+b^4)*(-1/
2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticP
i((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2
)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*
x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+2*a^2*(a^4-2*a^2*b^2+b^
4)/b^4*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*co
s(d*x+c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)...
```

3.70.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.70.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

3.70.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin\left(c+d\,x\right))^{7/2}}{(a+b\,\cos\left(c+d\,x\right))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2, x)
```

3.71 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

3.71.1 Optimal result

Integrand size = 25, antiderivative size = 404

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx &= -\frac{3ae^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d} \\ &+ \frac{3ae^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}d} \\ &+ \frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &+ \frac{3a^2e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\ &- \frac{3e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3/2*a*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2
))/b^(5/2)/(-a^2+b^2)^(1/4)/d+3/2*a*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))
^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(1/4)/d+e*(e*sin(d*x+c
))^(3/2)/b/d/(a+b*cos(d*x+c))-3/2*a^2*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-
(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*
sin(d*x+c))^(1/2)-3/2*a^2*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^
(1/2)+3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/si
n(d*x+c)^(1/2)
```

3.71.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.90 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.91

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{(e\sin(c+dx))^{5/2}\left(8b^{3/2}\csc(c+dx)+\frac{\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((e*Sin[c + d*x])^(5/2)*(8*b^(3/2)*Csc[c + d*x] + ((a + b*Sqrt[Cos[c + d*x
]^2])*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[S
in[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) +
 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/((a^2 - b^2)*Sin[c + d*x]^(5/2))))/(8
*b^(5/2)*d*(a + b*Cos[c + d*x]))
```

3.71.3 Rubi [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.97, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3172, 25, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{3e^2\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2b}+\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\int \sqrt{e\sin(c+dx)}dx}{b}-\frac{a\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\int \sqrt{e\sin(c+dx)}dx}{b}-\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{a\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{a\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$\Big\downarrow$ 3119

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b}$$

$\Big\downarrow$ 3180

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \cdots}{\cdots}\right.}{b}\right.}{2b}$$

$\Big\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \cdots}{\cdots}\right.}{b}\right.}{2b}$$

$\Big\downarrow$ 827

3.71. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

$$3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-a\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\cdots}{\cdots}\cdots\right)}{b}\cdots\right.}{2b}\cdots$$

$\downarrow$ 218

$$3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{\cdots}{\sqrt{e\sin(c+dx)}\left(\sqrt{\cdots}\cdots\right)}}{\cdots}\cdots\right)}{b}\cdots\right.}{2b}\cdots$$

$\downarrow$ 221

3.71. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\text{arct}\cdots}{2b\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots\right.}{2b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\text{arct}\cdots}{2b\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots\right.}{2b}$$

$\downarrow$ 3286

3.71. $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2}\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{2b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{a\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2}\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{2b}$$

$\downarrow$ 3284

3.71. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{3e^2\left(\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d} + \frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-}}\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin}}\right)}{b}\right)}{2b}$$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(e*(e*Sin[c + d*x])^(3/2))/(b*d*(a + b*Cos[c + d*x])) - (3*e^2*((2*Ellipti
cE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]]) -
 (a*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2
*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*
x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Ellipti
cPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]))))/b))/(2*b)
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

3.71. $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1667 vs. $2(436) = 872$.

Time = 15.83 (sec) , antiderivative size = 1668, normalized size of antiderivative = 4.13

method	result	size
default	Expression too large to display	1668

input
```
int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*e^3*a*b*(-1/2*(e*sin(d*x+c))^(3/2)/b^2/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
+3/16/b^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*s
in(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2
-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2
)-1)))+1/4*e^3*a^2*(3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2
)^(1/2)),1/2*2^(1/2))*b^2-3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2
+b^2)^(1/2))*b,1/2*2^(1/2))*b^2-12*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(5/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^3+6*(1
-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-si
n(d*x+c))^(1/2),1/2*2^(1/2))*b^3+3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1
/2)),1/2*2^(1/2))*b^3+3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(5/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^
(1/2))*b^3+3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),
1/2*2^(1/2))*a^2-3*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+...
```

3.71.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.71.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.71.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin\left(c+d\,x\right))^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2, x)
```

3.72 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

3.72.1 Optimal result

Integrand size = 25, antiderivative size = 418

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx &= \frac{ae^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} \\
&+ \frac{ae^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} - \frac{e^2\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a^2e^2\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{a^2e^2\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}
\end{aligned}
$$

output
```
1/2*a*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(3/2)/(-a^2+b^2)^(3/4)/d+1/2*a*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/d+e^2*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(1/2)-1/2*a^2*
e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/b^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/2*a^2*e^2*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/b^2/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+e*(e*sin(d*x+c))
^(1/2)/b/d/(a+b*cos(d*x+c))
```

3.72.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.49 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.33

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{(e\sin(c+dx))^{3/2}\left(\csc(c+dx) - \frac{\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2a\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

output
```
((e*Sin[c + d*x])^(3/2)*(Csc[c + d*x] - ((a + b*Sqrt[Cos[c + d*x]^2])*((a*
(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2
*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[
Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b
*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)
) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[
c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]])/((a^2 -
 b^2 + b^2*Sin[c + d*x]^2)*(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin
[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -
1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b
^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)])*Sin[c + d*x]^2))))/Sin[c + d*x]^(3/2)))/(b*d*(a + b*Cos[c + d*x])
)
```

3.72.3 Rubi [A] (warning: unable to verify)

Time = 1.81 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.98, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 3172, 25, 3042, 25, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3172$$

$$\frac{e^2\int -\frac{\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 25$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}-\frac{e^2\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}-\frac{e^2\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2b}$$

$$\Big\downarrow \ 25$$

$$\frac{e^2\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\left(a-b\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3346$$

$$\frac{e^2\left(\frac{a\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3121$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3120$$

$$\frac{e^2\left(\frac{a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}\right)}{2b}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3181$$

3.72. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{e^2\left(\frac{a\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2\cdots}\right.}}{2\sqrt{b^2-a^2}}\cdots}{b}\cdots\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 266

$$\frac{e^2\left(\frac{a\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}\right)}{b}-\cdots\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 756

$$\frac{e^2\left(\frac{a\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\cdots\right.}{b}\cdots\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 218

3.72. $\int\frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

$$e^2 \left(\frac{a \left(-\frac{2be \left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}} - \frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}} \right)}{d} - \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}} - \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}(\cdots}}{2\cdots} \cdots \right)}{b} \cdots}{2b} \cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 221$$

$$e^2 \left(\frac{a \left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}} - \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}} - \frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}} - \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\cdots}{\sqrt[4]{b\cdots}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-\cdots\right.} \cdots \right)}{d} \cdots \right)}{b} \cdots}{2b} \cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.72. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

$$e^2\left(\frac{a\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\ldots}{\sqrt[4]{b\ldots}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-\ldots\right.}\ldots\right.}{d}\ldots\right.}{b}\ldots}{2b}\ldots\right.$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 3286

$$e^2\left(\frac{a\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\ldots\right.}{d}\ldots\right.}{b}\ldots}{2b}\ldots\right.$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 3042

3.72. $\int\frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

$$e^2\left(\frac{a\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\right.}{}\right.}{b}\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

$\downarrow$ 3284

$$e^2\left(\frac{a\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\,\text{Ellipti}}{d\sqrt{b^2-a^2}\left(\right.}\right.}{b}\right.}{2b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

3.72. $\int\frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

output
```
(e*Sqrt[e*Sin[c + d*x]])/(b*d*(a + b*Cos[c + d*x])) + (e^2*((-2*EllipticF[
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) + (a
*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/
(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*
x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*El
lipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) -
 (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*
x]])))/b))/(2*b)
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.72. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. $2(449) = 898$.

Time = 4.17 (sec) , antiderivative size = 1370, normalized size of antiderivative = 3.28

method	result	size
default	Expression too large to display	1370

input
```
int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(-1/4/b^2*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
+1/32/b^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d
*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*(1/b^2*(1-s
in(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(-3*a^2+b^2)/
b^2*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*E
llipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(
-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((
1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-2*a^2*(a^2-b^2)
/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(
d*x+c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+
c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^...
```

3.72.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.72.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.72.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{(e\,\sin(c+d\,x))^{3/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2, x)
```

3.73 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

3.73.1 Optimal result

Integrand size = 25, antiderivative size = 438

$$\begin{aligned}\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx &= \frac{a\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} - \frac{a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d}\\
&\quad+\frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{a^2e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2b\left(a^2-b^2\right)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&\quad+\frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}}\\
&\quad-\frac{b(e\sin(c+dx))^{3/2}}{\left(a^2-b^2\right)de(a+b\cos(c+dx))}\end{aligned}$$

output
```
-b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))+1/2*a*arctan(b^(1/2
)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(5/4)/
d/b^(1/2)-1/2*a*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1
/2))*e^(1/2)/(-a^2+b^2)^(5/4)/d/b^(1/2)-1/2*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b-(-a^2
+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-1/2*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+(-a^2+b^2)^
(1/2))/(e*sin(d*x+c))^(1/2)-(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c)
)^(1/2)/(a^2-b^2)/d/sin(d*x+c)^(1/2)
```

3.73.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 13.70 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \frac{b\sin(c+dx)\sqrt{e\sin(c+dx)}}{(-a^2+b^2)\,d(a+b\cos(c+dx))}$$

$$+\frac{\sqrt{e\sin(c+dx)}\left(\frac{\cos^2(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
(b*Sin[c + d*x]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)*d*(a + b*Cos[c + d*x])
) + (Sqrt[e*Sin[c + d*x]]*((Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*
(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*
ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[S
qrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*
Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4
, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(
a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c + d
*x])*(1 - Sin[c + d*x]^2)) + (4*a*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 -
 ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (
(1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
 d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*Appell
F1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Si
n[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a +
 b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(2*(a - b)*(a + b)*d*Sqrt[Sin
[c + d*x]])
```

3.73.3 Rubi [A] (warning: unable to verify)

Time = 1.84 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.92, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3173, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{(2a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{a^2-b^2}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(2a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3346$$

$$\frac{a\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx+\int \sqrt{e\sin(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\int \sqrt{e\sin(c+dx)}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3121$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3119$$

$$\frac{a\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3180$$

$$\frac{a\left(-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 266$$

3.73. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

$$\frac{a\left(-\frac{2be\int \frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 827$$

$$\frac{a\left(-\frac{2be\left(\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 218$$

$$\frac{a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{}{\sqrt{e\sin(c+d}}}{}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 221$$

3.73. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

$$a\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt[4]{}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{}}\right.}{d}\right.$$

$$\overline{\qquad 2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$a\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt[4]{}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{}}\right.}{d}\right.$$

$$\overline{\qquad 2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3286

$$a\left(-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+}{\sqrt[4]{b^2-a}}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.$$

$$\overline{\qquad 2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 3042

3.73. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

$$a\left(-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+}{\sqrt[4]{b^2-a}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-}}\right.}{}\right.$$

$$\overline{\qquad 2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 3284

$$a\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+d}{}\right.$$

$$\overline{\qquad 2\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))) + ((2
*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*
x]]) + a*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4
)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c
 + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d +
(a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*El
lipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/(2*(a^2 - b^2)
)
```

3.73. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.73. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.73. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. 2(471) = 942.

Time = 4.09 (sec) , antiderivative size = 1306, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1306

input
```
int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(1/4*(e*sin(d*x+c))^(3/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2
*e^2+a^2*e^2)+1/32/(a^2*e^2-b^2*e^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)
*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+
(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e*
(1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b^2*(1-sin(d*x+c))^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))+2*a^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2
)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/8/(a^
2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-si...
```

3.73.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.73.6 Sympy [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin{(c+dx)}}}{(a+b\cos{(c+dx)})^2}\,dx$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x))**2, x)
```

3.73.7 Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin{(dx+c)}}}{(b\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.73.8 Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e\sin{(dx+c)}}}{(b\cos{(dx+c)}+a)^2}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2,x)
```

output
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2, x)
```

3.74 $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

3.74.1 Optimal result

Integrand size = 25, antiderivative size = 445

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx &= -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}} - \frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}}\\
&\quad - \frac{\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{e\sin(c+dx)}}\\
&\quad + \frac{3a^2\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&\quad + \frac{3a^2\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&\quad - \frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3/2*a*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/
2)/(-a^2+b^2)^(7/4)/d/e^(1/2)-3/2*a*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(
-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/d/e^(1/2)+(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi
+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-3/2*a
^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticP
i(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/(a^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/2*a^2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/
2)/(a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-b*(e*sin(
d*x+c))^(1/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))
```

3.74.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.67 (sec) , antiderivative size = 1182, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = -\frac{b\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{\sqrt{\sin(c+dx)}\left(-\frac{2b\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
-((b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]
])) + (Sqrt[Sin[c + d*x]]*((-2*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*
x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)
^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
 b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d
*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c
 + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4,
1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d
*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2))))))/((a + b*Cos[c + d*x])*(1 - Sin
[c + d*x]^2)) + (4*a*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8
 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqr
t[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + ...
```

3.74.3 Rubi [A] (warning: unable to verify)

Time = 1.91 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.95, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3173, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{2a-b\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a^2-b^2}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2a-b\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{2a+b\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3346$$

$$\frac{3a\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx-\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3121$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3120$$

$$\frac{3a\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

$$\Big\downarrow \ 3181$$

$$\frac{3a\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\right)}}{2\sqrt{b^2-a^2}}\right)}{2(a^2-b^2)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}$$

3.74. $$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$

$$\downarrow \ 266$$

$$\frac{3a\left(-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\downarrow \ 756$$

$$\frac{3a\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\downarrow \ 218$$

$$\frac{3a\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+}}}{}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\downarrow \ 221$$

3.74. $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

$$\frac{3a\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt{\cdots}\right.}{2\sqrt{b}e^{3/2}\left(\cdots\right.}\right.}{d}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 3042

$$\frac{3a\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\sqrt{\cdots}\right.}{2\sqrt{b}e^{3/2}\left(\cdots\right.}\right.}{d}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 3286

$$\frac{3a\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c\cdots}{\sqrt[4]{b^2-\cdots}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)\cdots}\right.}{\cdots}\right.}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 3042

3.74. $\displaystyle\int\frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

$$\frac{3a\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c\cdots}{\sqrt[4]{b^2-\cdots}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)\cdots}\cdots\right.}{\cdots}\cdots\right)}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

$\big\downarrow$ 3284

$$\frac{3a\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+\cdots}}{d\sqrt{\cdots}}\cdots\right)}{2\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
-((b*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))) + ((-2*
EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x
]]) + 3*a*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2
)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*S
in[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/
d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqr
t[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*S
in[c + d*x]])))/(2*(a^2 - b^2))
```

3.74. $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.74. $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. 2(476) = 952.

Time = 4.31 (sec) , antiderivative size = 1280, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	1280

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-4*a*b*e^3*(1/4*(e*sin(d*x+c))^(1/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2
*e^2+a^2*e^2)+3/32/(a^2*e^2-b^2*e^2)^2*(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(
ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e
^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(1/2
/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/(-a^2+b^2)
^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x
+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+2*a^2*(1/2*b^2/e/a^2/(a^2
-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a
^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5
/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b
)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/
4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2...
```

3.74.5 Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx=\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*sin(d*x + c))/((b^2*e*cos(d*x + c)^2 + 2*a*b*e*cos(d*x + c
) + a^2*e)*sin(d*x + c)), x)
```

3.74.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{e\sin{(c+dx)}}\,(a+b\cos{(c+dx)})^2}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2), x)
```

3.74.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.74.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx=\int \frac{1}{(b\cos{(dx+c)}+a)^2\sqrt{e\sin{(dx+c)}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x)
```

3.75 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

3.75.1 Optimal result

Integrand size = 25, antiderivative size = 507

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx &= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} \\
&- \frac{5ab^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} \\
&- \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2+3b^2)\cos(c+dx)}{(a^2-b^2)^2\,de\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&- \frac{5a^2b\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
&- \frac{(2a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)^2de^2\sqrt{\sin(c+dx)}}\end{aligned}$$

output
```
5/2*a*b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(9/4)/d/e^(3/2)-5/2*a*b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(3/2)-b/(a^2-b^2)/d/e
/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+(5*a*b-(2*a^2+3*b^2)*cos(d*x+c))/(a
^2-b^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/
(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b-(-a^2+b^
2)^(1/2))/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(
-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b+(-a^2+b^2)^(
1/2))/(e*sin(d*x+c))^(1/2)+(2*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*
(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e^2/sin(d*x+c)^(1/2)
```

3.75.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.54 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{12\left(-6a^2b+b^3+4a\left(a^2-b^2\right)\cos(c+dx)+b\left(2a^2+3b^2\right)\cos(2(c+dx))\right)}{\left(a^2-b^2\right)^2\sqrt{\sin(c+dx)}}+\frac{\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(\frac{\left(2a^2+3b^2\right)\sec(c+dx)}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
-1/24*(Sin[c + d*x]^(3/2)*((12*(-6*a^2*b + b^3 + 4*a*(a^2 - b^2)*Cos[c + d
*x] + b*(2*a^2 + 3*b^2)*Cos[2*(c + d*x)]))/((a^2 - b^2)^2*Sqrt[Sin[c + d*x
]]) + (Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*(((2*a^2 + 3*b^2)*Sec[c +
 d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[S
in[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) +
 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(Sqrt[b]*(-a^2 + b^2)) + (48*a*(a^2 +
 4*b^2)*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(
-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a
^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt
[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b
^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2))))/Sqr
t[Cos[c + d*x]^2]))/((a - b)^2*(a + b)^2)))/(d*(a + b*Cos[c + d*x])*(e*Sin
[c + d*x])^(3/2))
```

3.75.3 Rubi [A] (warning: unable to verify)

Time = 2.45 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {3042, 3173, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{2a-3b\cos(c+dx)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{a^2-b^2}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2a-3b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2a+3b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{\left(2a\left(a^2+4b^2\right)+b\left(2a^2+3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\left(2a\left(a^2+4b^2\right)+b\left(2a^2+3b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2a\left(a^2+4b^2\right)+b\left(2a^2+3b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3346$$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{(2a^2+3b^2)\int \sqrt{e\sin(c+dx)}dx+5ab^2\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.75. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{(2a^2+3b^2)\int\sqrt{e\sin(c+dx)}dx+5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\big\downarrow$ 3121

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{\frac{\left(2a^2+3b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\big\downarrow$ 3042

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{\frac{\left(2a^2+3b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\big\downarrow$ 3119

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2\left(2a^2+3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\big\downarrow$ 3180

$$\frac{\frac{2\left(5ab-\left(2a^2+3b^2\right)\cos(c+dx)\right)}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{\cdots}{\sqrt{e\sin(c\cdots}}\cdots}{\cdots}\cdots\right)\cdots}{e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\big\downarrow$ 266

3.75. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{\cdots}{\sqrt{e\sin(c\cdots}}\cdots}{\cdots}\cdots\right)}{e^2(a^2-b^2)}\cdots}{2\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 827

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{\cdots}{\sqrt{e\sin(c\cdots}}\cdots}{\cdots}\cdots\right)}{e^2\cdots}\cdots}{2\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 218

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2-\cdots}\right.}\cdots}{2b}\cdots\right)}{e^2(a^2-b^2\cdots}\cdots}{2\left(a^2-b^2\right)}\cdots$$

$$\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 221

3.75. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{}}{\sqrt[4]{}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{}}\cdots\right.}{\cdots}\right.}{e^2(a^2-b^2)}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{}}{\sqrt[4]{}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{}}\cdots\right.}{\cdots}\right.}{e^2(a^2-b^2)}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3286

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}\cdots\right.}{e^2(a^2\cdots}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\Big\downarrow$ 3042

3.75. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}\right.}{e^2(a^2}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

$\downarrow$ 3284

$$\frac{\frac{2(5ab-(2a^2+3b^2)\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{5ab^2\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}(c\right.}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}\right.}{e^2(a^2}}{2(a^2-b^2)}$$

$$\frac{b}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
-(b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])) + ((2*(5*
a*b - (2*a^2 + 3*b^2)*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]
) - ((2*(2*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(d*Sqrt[Sin[c + d*x]]) + 5*a*b^2*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Si
n[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - A
rcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2
 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sq
rt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Si
n[c + d*x]])))/((a^2 - b^2)*e^2))/(2*(a^2 - b^2))
```

3.75. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.75. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. 2(536) = 1072.

Time = 4.67 (sec) , antiderivative size = 2002, normalized size of antiderivative = 3.95

method	result	size
default	Expression too large to display	2002

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(-b^2/e^4/(a-b)^2/(a+b)^2*(1/4*(e*sin(d*x+c))^(3/2)/(-b^2*cos(
d*x+c)^2*e^2+a^2*e^2)+5/32/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*si
n(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-
b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^
2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)-1)))-1/e^4/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2))-1/4/e*
a^2*(5*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^
(1/2))*a^2*b-5*(-a^2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)*EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*
b,1/2*2^(1/2))*a^2*b-2*a^2*b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+5*(-a^2+b^2)
^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*Ellipt
icPi((1-sin(d*x+c))^(1/2),-b/(-b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3-5*(-a^
2+b^2)^(1/2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*
EllipticPi((1-sin(d*x+c))^(1/2),1/(b+(-a^2+b^2)^(1/2))*b,1/2*2^(1/2))*b^3+
8*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticE((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2*b^2-4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2...
```

3.75.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.75.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.75.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x)
```

3.76 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

3.76.1 Optimal result

Integrand size = 25, antiderivative size = 530

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx =
&-\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}}-\frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}}\\
&-\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\\
&+\frac{7ab-(2a^2+5b^2)\cos(c+dx)}{3\left(a^2-b^2\right)^2de(e\sin(c+dx))^{3/2}}\\
&+\frac{(2a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{3\left(a^2-b^2\right)^2de^2\sqrt{e\sin(c+dx)}}\\
&-\frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}\\
&-\frac{7a^2b^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}
\end{aligned}$$

output
```
-7/2*a*b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2
))/(-a^2+b^2)^(11/4)/d/e^(5/2)-7/2*a*b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c)
)^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(11/4)/d/e^(5/2)-b/(a^2-b^2)/
d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/3*(7*a*b-(2*a^2+5*b^2)*cos(d*x
+c))/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(3/2)-1/3*(2*a^2+5*b^2)*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi
+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(e*sin(d*x+c))^(1/2)
+7/2*a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*si
n(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c
))^(1/2)+7/2*a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(
1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*s
in(d*x+c))^(1/2)
```

3.76.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.81 (sec) , antiderivative size = 1257, normalized size of antiderivative = 2.37

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \frac{\left(\frac{b^3}{(a^2-b^2)^2(a+b\cos(c+dx))} - \frac{2(-2ab+a^2\cos(c+dx)+b^2\cos(c+dx))\csc^2(c+dx)}{3(a^2-b^2)^2}\right.}{d(e\sin(c+dx))^{5/2}}$$

$$+\frac{\sin^{\frac{5}{2}}(c+dx)\left(\frac{2(2a^2b+5b^3)\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a}\right.\right.}{}\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
((b^3/((a^2 - b^2)^2*(a + b*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x]
 + b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^2))*Sin[c + d*x]^3)/(d
*(e*Sin[c + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(2*a^2*b + 5*b^3)*Cos[c
 + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a
^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c
 + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*S
qrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b
^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[
c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2
)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^3 - 16*a*b^2)*
Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*Ar
cTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt
[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + ...
```

3.76.3 Rubi [A] (warning: unable to verify)

Time = 2.42 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.97, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3173, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{2a-5b\cos(c+dx)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{a^2-b^2}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2a-5b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2a+5b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}-\frac{2\int -\frac{2a\left(a^2-8b^2\right)+b\left(2a^2+5b^2\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2(a^2-b^2)}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{2a\left(a^2-8b^2\right)+b\left(2a^2+5b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2a\left(a^2-8b^2\right)-b\left(2a^2+5b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3346$$

$$\frac{\frac{(2a^2+5b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21ab^2\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.76. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{(2a^2+5b^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{\frac{\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2(a^2-b^2)}+\frac{2(7ab-(2a^2+5b^2)\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 3181$$

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-\cdots\right)}\cdots}{2\sqrt{b^2-a^2}}\cdots\right)}{3e^2(a^2-b^2)}\cdots}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \; 266$$

3.76. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 756

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right.}{d}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 218

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int\frac{\cdots}{\sqrt{e\sin(\cdots}}}{\cdots}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}\left(a+b\cos(c+dx)\right)}$$

$\downarrow$ 221

3.76. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\ldots\ 2be\ldots\right.}{3e^2(a^2-b^2)}$$

$$2\left(a^2-b^2\right)$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\ldots\ 2be\ldots\right.}{3e^2(a^2-b^2)}$$

$$2\left(a^2-b^2\right)$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3286

$$\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx\ldots\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}\right.}{3e^2(a^2-b^2)}$$

$$2\left(a^2-b^2\right)$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

3.76. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{\frac{2\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21ab^2\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\right.}{d\sqrt{b^2-a^2}\left(b-\sqrt{}\right.}\right.}{3e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}$$

input
```
Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
-(b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))) + ((2*(
7*a*b - (2*a^2 + 5*b^2)*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])
^(3/2)) + ((2*(2*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - 21*a*b^2*((-2*b*e*(-1/2*ArcTan[(Sqrt[b
]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^
(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqr
t[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2
+ b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b -
 Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2
]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/(3*(a^2 - b^2)*e^2))/(2
*(a^2 - b^2))
```

3.76. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.76. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. 2(556) = 1112.

Time = 6.30 (sec) , antiderivative size = 1474, normalized size of antiderivative = 2.78

method	result	size
default	Expression too large to display	1474

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-4*e^3*a*b*(-1/e^4/(a-b)^2/(a+b)^2*b^2*(1/4*(e*sin(d*x+c))^(1/2)/(-b^2*co
s(d*x+c)^2*e^2+a^2*e^2)+7/32*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2
^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^
(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^
2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/3/e^4/(a^2-b^2)^2/(e*sin(d*x
+c))^(3/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*(1/3*(-a^2-b^2)/(a^2-b^2
)^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((1-sin(d*x+c))^(1/2),
1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+2*a^2*b^2/(a-b)/(a+b)*(1/2*b^2/e/a
^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4
/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(
1/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^
(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1
/2))+1/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^
2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/...
```

3.76.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.76.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.76.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\sin(c+dx))^{5/2}\,(a+b\cos(c+dx))^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x)
```

3.77 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

3.77.1 Optimal result

Integrand size = 25, antiderivative size = 590

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx &= \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}} \\ &- \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}} \\ &- \frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} \\ &+ \frac{9ab-(2a^2+7b^2)\cos(c+dx)}{5\left(a^2-b^2\right)^2de(e\sin(c+dx))^{5/2}} - \frac{3(15ab^3+(2a^4-10a^2b^2-7b^4)\cos(c+dx))}{5\left(a^2-b^2\right)^3de^3\sqrt{e\sin(c+dx)}} \\ &+ \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\ &+ \frac{9a^2b^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{2\left(a^2-b^2\right)^3\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\ &- \frac{3(2a^4-10a^2b^2-7b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5\left(a^2-b^2\right)^3de^4\sqrt{\sin(c+dx)}}\end{aligned}$$

output
```
9/2*a*b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(13/4)/d/e^(7/2)-9/2*a*b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))
^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(7/2)-b/(a^2-b^2)/d
/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/5*(9*a*b-(2*a^2+7*b^2)*cos(d*x+
c))/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(5/2)-3/5*(15*a*b^3+(2*a^4-10*a^2*b^2-7
*b^4)*cos(d*x+c))/(a^2-b^2)^3/d/e^3/(e*sin(d*x+c))^(1/2)-9/2*a^2*b^3*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/
2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^
2-b^2)^3/d/e^3/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-9/2*a^2*b^3*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/
2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^
2-b^2)^3/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/5*(2*a^4-10*a^2
*b^2-7*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2
)^3/d/e^4/sin(d*x+c)^(1/2)
```

3.77.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.95 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \frac{\sin^4(c+dx)\left(-\frac{2(20ab^3+3a^4\cos(c+dx)-15a^2b^2\cos(c+dx)-8b^4\cos(c+dx))\,c}{5(a^2-b^2)^3}\right.}{d(e\,s}$$

$$-\frac{3\sin^{\frac{7}{2}}(c+dx)\left(\frac{(2a^4b-10a^2b^3-7b^5)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\mathrm{lo}\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(Sin[c + d*x]^4*((-2*(20*a*b^3 + 3*a^4*Cos[c + d*x] - 15*a^2*b^2*Cos[c + d
*x] - 8*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^3) - (2*(-2*a*b + a
^2*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c + d*x]^3)/(5*(a^2 - b^2)^2) - (b
^5*Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x]))))/(d*(e*Sin[c + d*x]
)^(7/2)) - (3*Sin[c + d*x]^(7/2)*(((2*a^4*b - 10*a^2*b^3 - 7*b^5)*Cos[c +
d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2]
+ Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]])
+ 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^
2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12
*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a
^5 - 10*a^3*b^2 - 22*a*b^4)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I
)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]
 + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4
, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c...
```

3.77.3 Rubi [A] (warning: unable to verify)

Time = 3.12 (sec) , antiderivative size = 583, normalized size of antiderivative = 0.99, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3173, 27, 3042, 3345, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{2a-7b\cos(c+dx)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{a^2-b^2}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2a-7b\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2a+7b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3345}$$

$$\frac{\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}-\frac{2\int -\frac{3\left(2a\left(a^2-4b^2\right)+b\left(2a^2+7b^2\right)\cos(c+dx)\right)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2(a^2-b^2)}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \frac{2a\left(a^2-4b^2\right)+b\left(2a^2+7b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3\int \frac{2a\left(a^2-4b^2\right)-b\left(2a^2+7b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 3345}$$

$$\frac{\frac{3\left(-\frac{2\int \frac{\left(2a\left(a^4-5b^2a^2-11b^4\right)+b\left(2a^4-10b^2a^2-7b^4\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c}{5de(a^2-b^2)(e\sin(c+dx}}{2(a^2-b^2)}-\frac{b}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

3.77. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

$$\frac{\frac{3\left(-\frac{\int \frac{\left(2a\left(a^4-5b^2a^2-11b^4\right)+b\left(2a^4-10b^2a^2-7b^4\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+}{5de(a^2-b^2)(e\sin(c+dx)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2a\left(a^4-5b^2a^2-11b^4\right)+b\left(2a^4-10b^2a^2-7b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7}{5de(a^2-b^2)(e}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3346

$$\frac{\frac{3\left(-\frac{\left(2a^4-10a^2b^2-7b^4\right)\int\sqrt{e\sin(c+dx)}dx-15ab^4\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}-$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{\left(2a^4-10a^2b^2-7b^4\right)\int\sqrt{e\sin(c+dx)}dx-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3121

3.77. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

$$\frac{\frac{3\left(-\frac{\frac{\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)}{5de(a^2-b^2)(e\sin(}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3\left(-\frac{\frac{\left(2a^4-10a^2b^2-7b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7b^2)}{5de(a^2-b^2)(e\sin(}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2(a^2-b^2)}+\frac{2(9ab-(2a^2+7}{5de(a^2-b^2)(e}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3180}$$

$$\frac{\frac{3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.}{5e^2(a^2-b^2)}}{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{266}$$

3.77. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{5e^2(a^2-b^2)}$$

$$\overline{2\left(a^2-b^2\right)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 827

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(\right.}}{2b}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{5e^2(a^2-b}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 218

3.77. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int -}{\sqrt{}}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{\qquad 5e^2(a^2-b^2)}$$

$$\overline{\qquad 2}$$

$$\frac{b}{de\,(a^2-b^2)\,(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 221

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{\qquad 5e^2(a^2-b^2)}$$

$$\overline{\qquad 2\,(a^2}$$

$$\frac{b}{de\,(a^2-b^2)\,(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

3.77. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{5e^2(a^2-b^2)}$$

$$2\,(a^2$$

$$\frac{b}{de\,(a^2-b^2)\,(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3286

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\right.}}{2b\sqrt{e\sin(c}}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{5e^2(a^2-b^2)}$$

$$\frac{b}{de\,(a^2-b^2)\,(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3042

3.77. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\right.}}{2b\sqrt{e\sin(c}}\right.}{e^2\left(a^2-b^2\right)}\right.}{5e^2(a^2-b^2)}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

$\downarrow$ 3284

$$\frac{2\left(9ab-\left(2a^2+7b^2\right)\cos(c+dx)\right)}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}+\frac{3\left(-\frac{\frac{2\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15ab^4\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{2b^{3/2}\sqrt{e}\sqrt[4]{}}\right.}{d}\right.}{}\right.}{}$$

$$\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}$$

3.77. $\int\frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

input
```
Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
-(b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))) + ((2*(
9*a*b - (2*a^2 + 7*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])
^(5/2)) + (3*((-2*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Cos[c + d*x]))/
((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - ((2*(2*a^4 - 10*a^2*b^2 - 7*b^4)*
EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x
]]) - 15*a*b^4*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2
)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]
*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]))
)/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (
a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 -
b^2)*e^2)))/(5*(a^2 - b^2)*e^2))/(2*(a^2 - b^2))
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.77. $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.77.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. $2(612) = 1224$.

Time = 6.69 (sec) , antiderivative size = 1749, normalized size of antiderivative = 2.96

method	result	size
default	Expression too large to display	1749

input
```
int(1/(a+cos(d*x+c)*b)^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*e^3*a*b*(2*b^4/e^6/(a-b)^3/(a+b)^3*(1/4*(e*sin(d*x+c))^(3/2)/(-b^2*cos
(d*x+c)^2*e^2+a^2*e^2)+9/32/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*s
in(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2
-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)-1)))-2/5/e^4/(a+b)^2/(a-b)^2/(e*sin(d*x+c))^(5/2)+
4/e^6/(a-b)^3/(a+b)^3*b^2/(e*sin(d*x+c))^(1/2))-(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/e^3*(-1/5*(-a^2-b^2)/(a^2-b^2)^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
/sin(d*x+c)/(cos(d*x+c)^2-1)*(6*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(7/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-sin(d*
x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c
))^(1/2),1/2*2^(1/2))+6*sin(d*x+c)*cos(d*x+c)^4-8*cos(d*x+c)^2*sin(d*x+c))
+b^2*(3*a^2+b^2)/(a^2-b^2)^3*(2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+
c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))
^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)-2*a^
2*b^4/(a-b)^2/(a+b)^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+...
```

3.77.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.77.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.77.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^2\left(e\sin\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^2*(e*sin(d*x + c))^(7/2)), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2),x)
```

output
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2), x)
```

3.78 $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

3.78.1 Optimal result

Integrand size = 25, antiderivative size = 590

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx &= \frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}\\
&- \frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}d}\\
&- \frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&- \frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^7\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+ \frac{11a(45a^2-37b^2)\,e^6E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}\\
&- \frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d}\\
&+ \frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

output
```
11/8*(9*a^4-11*a^2*b^2+2*b^4)*e^(13/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)
/(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-11/8*(9*a^4-11*a^2*
b^2+2*b^4)*e^(13/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/
e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-11/60*e^5*(45*a^2-10*b^2-27*a*b*cos(d
*x+c))*(e*sin(d*x+c))^(3/2)/b^5/d+11/28*e^3*(9*a+2*b*cos(d*x+c))*(e*sin(d*
x+c))^(7/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c))^(11/2)/b/d/(a+b*co
s(d*x+c))^2+11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^7/d/(b-(-a^2+b^2)^(1/2
))/(e*sin(d*x+c))^(1/2)+11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^7/d/(b+(-a
^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-11/20*a*(45*a^2-37*b^2)*e^6*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^6/d/sin(d*x+c)^(1/2)
```

3.78.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.73 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.58

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{11(e\sin(c+dx))^{13/2}\left(\frac{(45a^3-37ab^2)\cos^2(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx}}{\sqrt[4]{a^2-b^2}}\right.\right.\right.}{}\right.}{} + \frac{\csc^6(c+dx)(e\sin(c+dx))^{13/2}\left(\frac{(-168a^2+65b^2)\sin(c+dx)}{42b^5}-\frac{19(a^3\sin(c+dx)-ab^2\sin(c+dx))}{4b^5(a+b\cos(c+dx))}+\frac{a^4\sin(c+dx)-2a^2b^2\sin(c+dx)-}{2b^5(a+b\cos(c+dx))}\right.}{d}$$

input
```
Integrate[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(11*(e*Sin[c + d*x])^(13/2)*(((45*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2
]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(
a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2
 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*Appe
llF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]
*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 +
 b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(18*a^2*b - 10*b^3)*
Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x
]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]]
)/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/
(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))
*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c +
d*x]^2])))/(40*b^5*d*Sin[c + d*x]^(13/2)) + (Csc[c + d*x]^6*(e*Sin[c + d*x
])^(13/2)*(((-168*a^2 + 65*b^2)*Sin[c + d*x])/(42*b^5) - (19*(a^3*Sin[c +
d*x] - a*b^2*Sin[c + d*x]))/(4*b^5*(a + b*Cos[c + d*x])) + (a^4*Sin[c +...
```

3.78.3 Rubi [A] (warning: unable to verify)

Time = 2.71 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.92, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{13/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3172}$$

$$\frac{11e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \; 3342$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(-\frac{e^2\int -\frac{(2b+9a\cos(c+dx))(e\sin(c+dx))^{5/2}}{2(a+b\cos(c+dx))}dx}{b^2}-\frac{e(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{7b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 27$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\int \frac{(2b+9a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{7b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(2b+9a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{7b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 3344$$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e^2\int -\frac{\left(2b\left(9a^2-5b^2\right)+a\left(45a^2-37b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{5b^2}+\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{7/2}}{7b^2d(a+b}\right.}{4b}$$

$$\Big\downarrow \; 27$$

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int\frac{\left(2b\left(9a^2-5b^2\right)+a\left(45a^2-37b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{5b^2}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{7/2}(9}{7b^2d(a+b\,\mathrm{co}}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2b\left(9a^2-5b^2\right)+a\left(45a^2-37b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5b^2}\right)}{2b^2}-\frac{e(e\sin(c+}{7b}\right.}{4b}$$

$\downarrow$ 3346

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{5b^2}\right)}{2b^2}-\frac{e(e}{}\right.}{4b}$$

$\downarrow$ 3042

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{5b^2}\right)}{2b^2}-\right.}{4b}$$

$\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}}{b}\right.}{5b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

3.78. $\int\frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{a\left(45a^2-37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}}{b}\right.}{5b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3119

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\int\frac{\sqrt{e\cos\left(c+dx\right.}}{a-b\sin\left(c+dx\right.}}{b}\right.}{5b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3180

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} - \frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{be\int\frac{\cdots}{b^2\sin^2(\cdots}}{\cdots}\cdots\right.}{\cdots}\right)}{\cdots}\right)}{2b^2\cdots}\cdots\right)}{\cdots}$$

↓ 266

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{2be\int\frac{}{b^2e^4\,\mathrm{si}}}{}\right.}{}\right.}{}\right.}{2b^2}\right.$$

$\downarrow$ 827

3.78. $\int\frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-\frac{11e^2\left(\frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{2be\left(\frac{\int \cdots}{be^2 s\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

$\downarrow$ 218

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2\left(e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d}-\frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{2be\left(\frac{\arctan}{2b^{3/}}\right.}{}\right.}{}\right.}{}\right.\right.$$

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

3.77. ↓ 221

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$11e^2 \left(\frac{e^2 \left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2 \left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\int \frac{}{\sqrt{e\sin(}}}{} \right.}{} \right.}{} \right.}{} \right.$$

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$\downarrow$ 3042

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$11e^2 \left(\frac{e^2 \left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2 \left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\int \frac{\cdots}{\sqrt{e\sin(\cdots}}}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right.$$

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$\downarrow$ 3286

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$11e^2 \left(e^2 \left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\sqrt{\sin(c+d}}{}\right.}{}\right.}{}\right.\right.$$

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow 3042$$

3.78. $\int \frac{(e \sin(c+dx))^{13/2}}{(a+b \cos(c+dx))^3} \, dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$11e^2 \frac{e^2\left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2\left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(-\frac{ae\sqrt{\sin(c+d}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

3.78. $\downarrow$ 3284

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$11e^2 \left(\frac{e^2 \left(\frac{2e(e\sin(c+dx))^{3/2}\left(5\left(9a^2-2b^2\right)-27ab\cos(c+dx)\right)}{15b^2d} - \frac{e^2 \left(\frac{2a\left(45a^2-37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{5\left(9a^4-11a^2b^2+2b^4\right)\left(- \frac{2be\left(\frac{\arctan}{2b^{3/}}\right.}{} \right.}{} \right.}{} \right.}{} \right.$$

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

input
```
Int[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(11/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (11*e^2*(-1/7
*(e*(9*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(b^2*d*(a + b*Cos[c +
 d*x])) + (e^2*((2*e*(5*(9*a^2 - 2*b^2) - 27*a*b*Cos[c + d*x])*(e*Sin[c +
d*x])^(3/2))/(15*b^2*d) - (e^2*((2*a*(45*a^2 - 37*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]]) - (5*(9*a^4 -
 11*a^2*b^2 + 2*b^4)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2
 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*S
qrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqr
t[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]
]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b)
)/(5*b^2)))/(2*b^2)))/(4*b)
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.78. $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

3.78.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2994 vs. $2(607) = 1214$.

Time = 100.32 (sec) , antiderivative size = 2995, normalized size of antiderivative = 5.08

method	result	size
default	Expression too large to display	2995

input
```
int((e*sin(d*x+c))^(13/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-1/21/b^6*(e*sin(d*x+c))^(3/2)*e^2*(3*b^2*cos(d*x+c)^2+42*a^2-17
*b^2)+e^4/b^6*(-1/8*(e*sin(d*x+c))^(3/2)*e^2*(-21*a^4*b^2*cos(d*x+c)^2+23*
a^2*b^4*cos(d*x+c)^2-2*b^6*cos(d*x+c)^2+17*a^6-15*a^4*b^2-2*a^2*b^4)/(-b^2
*cos(d*x+c)^2*e^2+a^2*e^2)^2+1/8*(99/8*a^4-121/8*a^2*b^2+11/4*b^4)/b^2/(e^
2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)
*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1
/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2
*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*e^7*a*(1/5/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*(100*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-78*(1-sin(d*x+c))^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/
2))*b^2-50*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*El
lipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2+39*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(
1/2))*b^2+6*b^2*cos(d*x+c)^4-6*b^2*cos(d*x+c)^2)+3*(7*a^4-10*a^2*b^2+3*b^4
)/b^6*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-
sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(...
```

3.78.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(13/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.78.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.78.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{13}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(13/2)/(b*cos(d*x + c) + a)^3, x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\ \sin\left(c+d\,x\right))^{13/2}}{\left(a+b\ \cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3, x)
```

3.79 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

3.79.1 Optimal result

Integrand size = 25, antiderivative size = 604

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d} \\
&- \frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d} \\
&+ \frac{3a(21a^2-13b^2)\,e^6\operatorname{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}} \\
&- \frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{9a(7a^4-9a^2b^2+2b^4)\,e^6\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^6\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&- \frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} \\
&+ \frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}
\end{aligned}
$$

output
```
-9/8*(7*a^4-9*a^2*b^2+2*b^4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/
(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d-9/8*(7*a^4-9*a^2*b^2
+2*b^4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(
1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d+9/20*e^3*(7*a+2*b*cos(d*x+c))*(e*sin(d*x
+c))^(5/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c))^(9/2)/b/d/(a+b*cos(
d*x+c))^2-3/4*a*(21*a^2-13*b^2)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d
*x+c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)+9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(co
s(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/b^6/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+9/8*a*(7*a^4-9*a^
2*b^2+2*b^4)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2)
)*sin(d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)
-3/4*e^5*(21*a^2-6*b^2-7*a*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

3.79.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 15.20 (sec) , antiderivative size = 2024, normalized size of antiderivative = 3.35

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(((2*a*Cos[c + d*x])/b^4 + (-a^2 + b^2)^2/(2*b^5*(a + b*Cos[c + d*x])^2) -
 (17*a*(a^2 - b^2))/(4*b^5*(a + b*Cos[c + d*x])) - Cos[2*(c + d*x)]/(5*b^3
))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d + (3*(e*Sin[c + d*x])^(11/2)*
((2*(25*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((
a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] +
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/
4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/(
(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d
*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a
^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]
^2)) + (2*(30*a^2*b - 16*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2]
)*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]]
)/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/
(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^...
```

3.79.3 Rubi [A] (warning: unable to verify)

Time = 2.74 (sec) , antiderivative size = 562, normalized size of antiderivative = 0.93, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3344, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{11/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{9e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\int\frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\int\frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \text{3342}$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(-\frac{e^2\int-\frac{(2b+7a\cos(c+dx))(e\sin(c+dx))^{3/2}}{2(a+b\cos(c+dx))}dx}{b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\int\frac{(2b+7a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\int\frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(2b+7a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3344}$$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e^2\int-\frac{2b\left(7a^2-3b^2\right)+a\left(21a^2-13b^2\right)\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}+\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{27}$$

3.79. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}dx$

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2b\left(7a^2-3b^2\right)+a\left(21a^2-13b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3b^2}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{}}{4b}$$

$\downarrow$ 3042

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\int\frac{2b\left(7a^2-3b^2\right)-a\left(21a^2-13b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3b^2}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{5b^2d(a+b\cos(c+dx))}\right)}{}}{4b}$$

$\downarrow$ 3346

$$\frac{\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{3b^2}\right)}{2b^2}\right)}{}}{4b}$$

$\downarrow$ 3042

3.79. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\right.}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

3.79. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\right.}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3120

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-\frac{9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\int\frac{}{\sqrt{e\cos\left(c+dx-\right.}}}{b}\right.}{3b^2}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3181

3.79. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{be\int\frac{\cdots}{\sqrt{e\sin(c+\cdots}}}{\cdots}\right.}{\cdots}\right)}{\cdots}\right)}{2b^2\cdots}\right.$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\int\frac{\cdots}{b^2e^4\sin\cdots}}{\cdots}\right.}{\cdots}\right)}{\cdots}\right)}{2b^2\cdots}\right.$$

$\downarrow$ 756

3.79. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2\left(e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - 3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\left(-\frac{\int}{\sqrt{b^2}}\right.}{}\right.\right.\right.\right.$$

$\downarrow$ 218

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2 \left(\frac{e^2 \left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\left(-\frac{\int \frac{\cdots}{\sqrt{b^2 \cdots}}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$\downarrow$ 221

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d}-\frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\int\frac{}{\sqrt{e\sin(c+}}}{}\right.}{}\right.}{}\right.}{2}\right.$$

3.79. $\int\frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

3.79. $\int \frac{(e \sin(c+dx))^{11/2}}{(a+b \cos(c+dx))^3} \, dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2 \left(\frac{e^2 \left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2 d} - \frac{e^2 \left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\int \frac{\cdots}{\sqrt{e\sin(c+\cdots}}}{\cdots} \cdots \right.}{\cdots} \right.}{\cdots} \right.}{2\cdots} \right.$$

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$\downarrow$ 3286

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2 \left(\frac{e^2 \left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\sqrt{\sin(c+dx)}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\downarrow 3042$$

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2\left(\frac{e^2\left(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{a\sqrt{\sin(c+dx)}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$$

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\downarrow 3284$$

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$9e^2 \Bigg(e^2 \Bigg(\frac{2e\sqrt{e\sin(c+dx)}\left(3\left(7a^2-2b^2\right)-7ab\cos(c+dx)\right)}{3b^2d} - \frac{e^2\left(\frac{2a\left(21a^2-13b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{3\left(7a^4-9a^2b^2+2b^4\right)\left(-\frac{2be\left(-\frac{\arctan}{2\sqrt{b}}\right.}{}\right.}{}\right.}{}$$

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

input
```
Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(9/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (9*e^2*(-1/5*(
e*(7*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(b^2*d*(a + b*Cos[c + d
*x])) + (e^2*((2*e*(3*(7*a^2 - 2*b^2) - 7*a*b*Cos[c + d*x])*Sqrt[e*Sin[c +
 d*x]])/(3*b^2*d) - (e^2*((2*a*(21*a^2 - 13*b^2)*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) - (3*(7*a^4 - 9*a^2
*b^2 + 2*b^4)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 +
 b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[
e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)
)))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin
[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Sin[c + d*x]])))/b))/(3*b^2)))/(2*b^2)))/(4*b)
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.79. $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

3.79.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2751 vs. 2(621) = 1242.

Time = 100.88 (sec) , antiderivative size = 2752, normalized size of antiderivative = 4.56

method	result	size
default	Expression too large to display	2752

input
```
int((e*sin(d*x+c))^(11/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-1/5/b^6*(e*sin(d*x+c))^(1/2)*e^2*(b^2*cos(d*x+c)^2+30*a^2-11*b^
2)+e^4/b^6*(-1/8*(e*sin(d*x+c))^(1/2)*e^2*(-19*a^4*b^2*cos(d*x+c)^2+21*a^2
*b^4*cos(d*x+c)^2-2*b^6*cos(d*x+c)^2+15*a^6-13*a^4*b^2-2*a^2*b^4)/(-b^2*co
s(d*x+c)^2*e^2+a^2*e^2)^2+9/64*(7*a^4-9*a^2*b^2+2*b^4)*(e^2*(a^2-b^2)/b^2)
^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(
e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)
+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^6*a*(1/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*(10*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*a^2-7*(1-sin(d*x+c))^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)
)*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c))+(21*a^4-30*a^2*b^2+9*b^4)/b^6*(-1/2/(
-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((
1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(
1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c
))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+(-15*a^6+33*a^4*b^2-21*...
```

3.79.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.79.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.79.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{11}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(11/2)/(b*cos(d*x + c) + a)^3, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\ \sin(c+d\,x))^{11/2}}{(a+b\ \cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3, x)
```

3.80 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

3.80.1 Optimal result

Integrand size = 25, antiderivative size = 498

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{7(5a^2-2b^2)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d}\\
&+\frac{7(5a^2-2b^2)\,e^{9/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}d}\\
&+\frac{7a(5a^2-2b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{7a(5a^2-2b^2)\,e^5\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{35ae^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}}\\
&+\frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

output
```
-7/8*(5*a^2-2*b^2)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1/4)/d+7/8*(5*a^2-2*b^2)*e^(9/2)*arctan
h(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2
)^(1/4)/d+7/12*e^3*(5*a+2*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d/(a+b*co
s(d*x+c))+1/2*e*(e*sin(d*x+c))^(7/2)/b/d/(a+b*cos(d*x+c))^2-7/8*a*(5*a^2-2
*b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/8*a*(5*a^2-2
*b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+35/4*a*e^4*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)
```

3.80.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.43 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.68

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^4(c+dx)(e\sin(c+dx))^{9/2}\left(\frac{2\sin(c+dx)}{3b^3}+\frac{11a\sin(c+dx)}{4b^3(a+b\cos(c+dx))}+\frac{-a^2\sin(c+dx)+b^2\sin(c}{2b^3(a+b\cos(c+dx)}\right.}{d}$$

$$-\frac{7(e\sin(c+dx))^{9/2}\left(\frac{5a\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*((2*Sin[c + d*x])/(3*b^3) + (11*a*S
in[c + d*x])/(4*b^3*(a + b*Cos[c + d*x])) + (-(a^2*Sin[c + d*x]) + b^2*Sin
[c + d*x])/(2*b^3*(a + b*Cos[c + d*x])^2)))/d - (7*(e*Sin[c + d*x])^(9/2)*
((5*a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sq
rt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*S
in[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2
*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c +
 d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]
^2)) + (4*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2
 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[
c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(
a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[
1 - Sin[c + d*x]^2])))/(8*b^3*d*Sin[c + d*x]^(9/2))
```

3.80.3 Rubi [A] (warning: unable to verify)

Time = 2.14 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{9/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3172}$$

$$\frac{7e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\int\frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\int\frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \text{3342}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(-\frac{e^2\int-\frac{(2b+5a\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int\frac{(2b+5a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2b+5a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{5a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{\left(5a^2-2b^2\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

3.80. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{7e^2\left(\frac{e^2\left(\frac{5a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{\left(5a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-$$

$\Big\downarrow$ 3121

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{7e^2\left(\frac{e^2\left(\frac{5a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\Big\downarrow$ 3042

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{7e^2\left(\frac{e^2\left(\frac{5a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$\Big\downarrow$ 3119

3.80. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}\right)}{2b^2}-\frac{e(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{3b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \; 3180$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2b}\right.}{b}\right.}{2b^2}}{4b}$$

$$\Big\downarrow \; 266$$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2b}\right.}{b}\right.}{2b^2}}{4b}$$

$$\Big\downarrow \; 827$$

3.80. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{\left(5a^2-2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b} - \frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx}}{2b}\right.}{d}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$$\Big\downarrow \; 218$$

3.80. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(5a^2-2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{}{\sqrt{e\sin}}}{}\right)}{b}\right)}{2b^2}\right)}{4b}$$

$\downarrow$ 221

3.80. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{(5a^2-2b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b} + \frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}\right.}{b}}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

3.80. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{\left(5a^2-2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b} + \frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b} \cdots\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3286

3.80. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$7e^2\frac{\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

3.80. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(5a^2-2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c\right.}}{2b\sqrt{e\sin(c+dx)}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3284

3.80. $\int\frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{7e^2\left(\frac{e^2\left(\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{\left(5a^2-2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\operatorname{EllipticP}}{bd\left(b-\sqrt{b^2}\right)}\right)}{b}\right)}{2b^2}\right)}{4b}$$

input
```
Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

3.80. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

output
```
(e*(e*Sin[c + d*x])^(7/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (7*e^2*(-1/3*(
e*(5*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(b^2*d*(a + b*Cos[c + d
*x])) + (e^2*((10*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/(b*d*Sqrt[Sin[c + d*x]]) - ((5*a^2 - 2*b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqr
t[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[
e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2
)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 +
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2
])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
 (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sq
rt[e*Sin[c + d*x]])))/b))/(2*b^2)))/(4*b)
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.80. $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2735 vs. $2(522) = 1044$.

Time = 98.27 (sec) , antiderivative size = 2736, normalized size of antiderivative = 5.49

method	result	size
default	Expression too large to display	2736

input
```
int((e*sin(d*x+c))^(9/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(1/3*(e*sin(d*x+c))^(3/2)/b^4-e^2/b^4*(-1/8*(e*sin(d*x+c))^(3/2)*
e^2*(-13*a^2*b^2*cos(d*x+c)^2+2*b^4*cos(d*x+c)^2+9*a^4+2*a^2*b^2)/(-b^2*co
s(d*x+c)^2*e^2+a^2*e^2)^2+1/8*(35/8*a^2-7/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(
1/4)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)*e^5*a*(-3/b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((1-sin(d*x+c))^
(1/2),1/2*2^(1/2))-EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)))-2*(5*a^2-3
*b^2)/b^4*(-1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin
(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1
+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+(11*a^4-14*a^2*b^2+3*b^4)/b^4*(1/2*b^2/
e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x
+c)^2+a^2)-1/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+...
```

3.80.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.80.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.80.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{9}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\ \sin(c+d\,x))^{9/2}}{(a+b\ \cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3, x)
```

3.81 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

3.81.1 Optimal result

Integrand size = 25, antiderivative size = 512

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx &= \frac{5(3a^2-2b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} \\
&+ \frac{5(3a^2-2b^2)\,e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} \\
&- \frac{15ae^4\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^4d\sqrt{e\sin(c+dx)}} \\
&+ \frac{5a(3a^2-2b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{5a(3a^2-2b^2)\,e^4\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^4\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} \\
&+ \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}\end{aligned}$$

output
```
5/8*(3*a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/d+5/8*(3*a^2-2*b^2)*e^(7/2)*arctanh
(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)
^(3/4)/d+1/2*e*(e*sin(d*x+c))^(5/2)/b/d/(a+b*cos(d*x+c))^2+15/4*a*e^4*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(e*sin(d*x+c))^(1/2)-5
/8*a*(3*a^2-2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))
^(1/2)-5/8*a*(3*a^2-2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^
(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin
(d*x+c))^(1/2)+5/4*e^3*(3*a+2*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^3/d/(a+
b*cos(d*x+c))
```

3.81.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.94 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.85

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^3(c+dx)(e\sin(c+dx))^{7/2}\left(7a^2+2b^2+9ab\cos(c+dx)+\frac{\cdots}{\frac{1}{9}\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sin\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^3*(e*Sin[c + d*x])^(7/2)*(7*a^2 + 2*b^2 + 9*a*b*Cos[c + d*x]
 + ((a + b*Cos[c + d*x])*(-6*b - 7*a*Cos[c + d*x] + 4*b*Cos[2*(c + d*x)])*
(8*(a + b) - 5*(3*a + 2*b)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[(c + d*x)/2]^2,
 ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2] + (3*a -
2*b)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*
x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2))/((Sqrt[Sec
[(c + d*x)/2]^2]*Sin[c + d*x]*Tan[(c + d*x)/2]*(-45*(3*a + 2*b)*AppellF1[1
/4, 1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b
)] + 18*(3*a - 2*b)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a +
 b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2 + (9*(3*a + 2*b)*(2*(a
 - b)*AppellF1[5/4, 1/2, 2, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d
*x)/2]^2)/(a + b)] + (a + b)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[(c + d*x)/2]^
2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2)/(a + b) + 9
*(3*a - 2*b)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan
[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (5*(3*a - 2*b)*(2*(a - b)*A
ppellF1[9/4, 1/2, 2, 13/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]
^2)/(a + b)] + (a + b)*AppellF1[9/4, 3/2, 1, 13/4, -Tan[(c + d*x)/2]^2, ((
-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^
2)/(a + b)))/9 + Cos[c + d*x]*(8*(a + b) - 5*(3*a + 2*b)*AppellF1[1/4, 1/2
, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*S...
```

3.81.3 Rubi [A] (warning: unable to verify)

Time = 2.23 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.94, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3342, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{5e^2\int -\frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\int \frac{\left(-e\cos\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \text{3342}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(-\frac{e^2\int -\frac{2b+3a\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \frac{2b+3a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \frac{2b-3a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{3a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{\left(3a^2-2b^2\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-5e^2\left(\frac{e^2\left(\frac{3a\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \textbf{3121}$$

$$\frac{\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-5e^2\left(\frac{e^2\left(\frac{3a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-5e^2\left(\frac{e^2\left(\frac{3a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}\right)}{2b^2}-\frac{e\sqrt{e\sin(c+dx)}(3a+2b\cos(c+dx))}{b^2d(a+b\cos(c+dx))}\right)}{4b}$$

$$\Big\downarrow \textbf{3181}$$

3.81. $\int\frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int\frac{}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2\right.}}}{2\sqrt{b^2}}\right.}{b}\right.}{2b^2}\right.$$

$$4b$$

$\downarrow$ 266

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+\right.}}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{2b^2}\right.$$

$$4b$$

$\downarrow$ 756

3.81. $\int\frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{\left(3a^2-2b^2\right)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}} - \frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin}}{2e\sqrt{b^2-a^2}}\right.}{d}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 218

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{(3a^2-2b^2)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}} - \frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d} - \frac{a\int -\cdots}{\sqrt{\cdots}}\right)}{b}\right)}{2b^2}\right)}{4b}$$

$\downarrow$ 221

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}d}{2\sqrt{b^2-a^2}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3042

3.81. $\int\frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{5e^2\Bigg(\frac{e^2\Bigg(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\Bigg(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}d}{2\sqrt{b^2-a^2}}}{b}}{2b^2}}{4b}$$

$\downarrow$ 3286

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}} - \frac{\left(3a^2-2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}} - \frac{a\sqrt{\sin(c+dx)}\int\frac{\cdots}{\sqrt{\sin(c+dx)}\left(b\sin\cdots\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(\cdots}\cdots\right)}{b}\cdots\right)}{2b^2}\cdots\right)}{4b\cdots}$$

$\downarrow$ 3042

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{(3a^2-2b^2)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{}{\sqrt{\sin(c+dx)}\left(b\sin\right.}}{2\sqrt{b^2-a^2}\sqrt{e\sin(}}\right.}{b}\right.}{2b^2}\right.}{4b}$$

$\downarrow$ 3284

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}-\frac{5e^2\left(\frac{e^2\left(\frac{6a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}-\frac{\left(3a^2-2b^2\right)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{Elli}}{d\sqrt{b^2-a}}\right)}{b}\right)}{2b^2}\right)}{4b}$$

input
```
Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

output
```
(e*(e*Sin[c + d*x])^(5/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (5*e^2*(-((e*(
3*a + 2*b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(b^2*d*(a + b*Cos[c + d*x]))
) + (e^2*((6*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*S
qrt[e*Sin[c + d*x]]) - ((3*a^2 - 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqr
t[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)
) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*
(-a^2 + b^2)^(3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2
]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt
[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b
+ Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b))/(2*b^2)))/(4*b)
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.81. $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.81.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2588 vs. $2(536) = 1072$.

Time = 98.09 (sec) , antiderivative size = 2589, normalized size of antiderivative = 5.06

method	result	size
default	Expression too large to display	2589

input
```
int((e*sin(d*x+c))^(7/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*((e*sin(d*x+c))^(1/2)/b^4-e^2/b^4*(-1/8*(e*sin(d*x+c))^(1/2)*e^2*
(-11*a^2*b^2*cos(d*x+c)^2+2*b^4*cos(d*x+c)^2+7*a^4+2*a^2*b^2)/(-b^2*cos(d*
x+c)^2*e^2+a^2*e^2)^2+5/64*(3*a^2-2*b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^
2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+
c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/
b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arct
an(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1
/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1))))-(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)*e^4*a*(-3/b^4*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c
))^(1/2),1/2*2^(1/2))+(-10*a^2+6*b^2)/b^4*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-
(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^
(1/2)/b),1/2*2^(1/2)))+1/b^4*(11*a^4-14*a^2*b^2+3*b^4)*(1/2*b^2/e/a^2/(a^2
-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a
^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2)...
```

3.81.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

3.81.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{7/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3, x)
```

3.82 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

3.82.1 Optimal result

Integrand size = 25, antiderivative size = 520

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{3(a^2-2b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d}\\
&+\frac{3(a^2-2b^2)\,e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d}\\
&-\frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{3a(a^2-2b^2)\,e^3\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^3\left(a^2-b^2\right)\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{3ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}}\\
&+\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3ae(e\sin(c+dx))^{3/2}}{4b\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-3/8*(a^2-2*b^2)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/d+3/8*(a^2-2*b^2)*e^(5/2)*arctanh(b^
(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5
/4)/d+1/2*e*(e*sin(d*x+c))^(3/2)/b/d/(a+b*cos(d*x+c))^2-3/4*a*e*(e*sin(d*x
+c))^(3/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))+3/8*a*(a^2-2*b^2)*e^3*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1
/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/(a^2
-b^2)/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/8*a*(a^2-2*b^2)*e^3*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos
(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/
b^3/(a^2-b^2)/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/4*a*e^2*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/sin(d*x+c)
^(1/2)
```

3.82.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 14.24 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.60

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\csc^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{\sin(c+dx)}{2b(a+b\cos(c+dx))^2}+\frac{3a\sin(c+dx)}{4b(-a^2+b^2)(a+b\cos(c+dx))}\right)}{d}$$

$$+\frac{3(e\sin(c+dx))^{5/2}\left(\frac{a\cos^2(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{\cdots}\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(Sin[c + d*x]/(2*b*(a + b*Cos[c + d
*x])^2) + (3*a*Sin[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d +
 (3*(e*Sin[c + d*x])^(5/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/
4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Lo
g[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1,
7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)
)*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c
+ d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[
1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
+ ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[
c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*App
ellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]
*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((
a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)*b*(a + b)*d*Sin
[c + d*x]^(5/2))
```

3.82.3 Rubi [A] (warning: unable to verify)

Time = 2.15 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.89, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3172, 25, 3042, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{3e^2\int -\frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx}{4b}+\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\sin\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \; 3343$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{(2b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{a^2-b^2}\right)}{4b}$$

$$\Big\downarrow \; 27$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{(2b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \; 3346$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\int\sqrt{e\sin(c+dx)}dx}{b}-\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \; 3121$$

3.82. $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{a\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{b\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{b}}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \text{ 3180}$$

$$\frac{\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{\cdots}{\sqrt{e\sin(c+\cdots}}}{\cdots}\right)}{\cdots}}{2(a^2-b^2)}\right)}{4b}$$

$$\Big\downarrow \text{ 266}$$

3.82. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(c}}}{}\right)}{}}{2(a^2-b^2)}\right)}{4b}$$

$\downarrow$ 827

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$\frac{3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{\left(a^2-2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{}{\sqrt{b^2-a^2}e-b}}{}\right)}{d}\right)}{}}{}\right)}{4b}$$

$\downarrow$ 218

3.82. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))} - \frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{(a^2-2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}} - \frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{}}{2b}\right)}{d}\right.}{}}{2(a^2-b^2}\right.$$

$$4b$$

↓ 221

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} -$$

$$3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))} - \frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}} - \frac{(a^2-2b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b} + \frac{ae\int\frac{}{\sqrt{e\sin(c+dx)}}}{}\right.}{}}{2(a^2-b^2)}\right.$$

$$4b$$

↓ 3042

3.82. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{}{\sqrt{e\sin(c+dx)}}\right.}{}}{2(a^2-b^2)}\right.$$

$$\overline{\qquad 4b \qquad}$$

$\downarrow$ 3286

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(}}{}\right.}{}}{2(a}\right.$$

$$\overline{\qquad 4b \qquad}$$

$\downarrow$ 3042

3.82. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(}{}\right.}{}}{2(a^2}\right.$$

$$4b$$

$\downarrow$ 3284

$$\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}-$$

$$3e^2\left(\frac{a(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}-\frac{\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{bd\sqrt{\sin(c+dx)}}-\frac{(a^2-2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right.}{d}\right.}{2(a^2}\right.$$

$$4b$$

3.82. $\int\frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

input
```
Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*(e*Sin[c + d*x])^(3/2))/(2*b*d*(a + b*Cos[c + d*x])^2) - (3*e^2*((a*(e*
Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])) - ((2*a*Ellipt
icE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(b*d*Sqrt[Sin[c + d*x]])
- ((a^2 - 2*b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b
^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[
e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]
)))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) +
 (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/b)/(2*
(a^2 - b^2))))/(4*b)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.82. $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(544) = 1088$.

Time = 98.23 (sec) , antiderivative size = 2612, normalized size of antiderivative = 5.02

method	result	size
default	Expression too large to display	2612

input
```
int((e*sin(d*x+c))^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(e^3*b*(-1/4*(e*sin(d*x+c))^(3/2)*e^2*(-5*a^2*b^2*cos(d*x+c)^2+2*b^4*cos(d
*x+c)^2+a^4+2*a^2*b^2)/b^2/(a^2-b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2+3/3
2*(a^2-2*b^2)/b^4/(a^2-b^2)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*(ln((e*sin(d
*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^3*a*(3/b^2*(-
1/2/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+
c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(1-sin(d*x+c))^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^
(1/2)/b),1/2*2^(1/2)))-(7*a^2-3*b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+
c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-1/2/a^2/(a^2-
b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+1/4/
a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1
/2))-3/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(...
```

3.82.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.82.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

3.82.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\ \sin\left(c+d\,x\right))^{5/2}}{\left(a+b\ \cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3, x)
```

3.83 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

3.83.1 Optimal result

Integrand size = 25, antiderivative size = 534

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx &= -\frac{(a^2+2b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\,(-a^2+b^2)^{7/4}\,d}\\
&-\frac{(a^2+2b^2)\,e^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{3/2}\,(-a^2+b^2)^{7/4}\,d}\\
&-\frac{ae^2\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4b^2\,(a^2-b^2)\,d\sqrt{e\sin(c+dx)}}\\
&+\frac{a(a^2+2b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2\,(a^2-b^2)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{a(a^2+2b^2)\,e^2\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b^2\,(a^2-b^2)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}-\frac{ae\sqrt{e\sin(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-1/8*(a^2+2*b^2)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/d-1/8*(a^2+2*b^2)*e^(3/2)*arctanh(b^
(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7
/4)/d+1/4*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2
-b^2)/d/(e*sin(d*x+c))^(1/2)-1/8*a*(a^2+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(a^2-
b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/8*a*(a^2+2*b^2)*e^2*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*
c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(
a^2-b^2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+1/2*e*(e*sin(
d*x+c))^(1/2)/b/d/(a+b*cos(d*x+c))^2-1/4*a*e*(e*sin(d*x+c))^(1/2)/b/(a^2-b
^2)/d/(a+b*cos(d*x+c))
```

3.83.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 1211, normalized size of antiderivative = 2.27

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{\left(\frac{1}{2b(a+b\cos(c+dx))^2}+\frac{a}{4b(-a^2+b^2)(a+b\cos(c+dx))}\right)\csc(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

$$- \frac{(e\sin(c+dx))^{3/2}\left(\frac{2a\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
((1/(2*b*(a + b*Cos[c + d*x])^2) + a/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x]
)))*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/d - ((e*Sin[c + d*x])^(3/2)*((2*a
*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[
2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*
Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[
Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b
*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b
^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] +
2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c +
d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (4*b*Cos[c + d*x
]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((
1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*
x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si...
```

3.83.3 Rubi [A] (warning: unable to verify)

Time = 2.25 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.90, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 3172, 25, 3042, 25, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 3172$$

$$\frac{e^2\int -\frac{\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}-\frac{e^2\int\frac{\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}-\frac{e^2\int-\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^2\int\frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\sqrt{e\cos\left(\frac{1}{2}(2c-\pi)+dx\right)}\left(a-b\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^2}dx}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3343}$$

$$\frac{e^2\left(-\frac{\int-\frac{2b-a\cos(c+dx)}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a^2-b^2}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\left(\frac{\int\frac{2b-a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{\int\frac{2b+a\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3346}$$

$$\frac{e^2\left(\frac{\frac{(a^2+2b^2)\int\frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{b}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

3.83. $\int\frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{e^2\left(\frac{\frac{\left(a^2+2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}}dx}{b}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3121

$$\frac{e^2\left(\frac{\frac{\left(a^2+2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{e^2\left(\frac{\frac{\left(a^2+2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}}dx}{b\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3120

$$\frac{e^2\left(\frac{\frac{\left(a^2+2b^2\right)\int\frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{a\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}\right)}{4b}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3181

3.83. $\int\frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{be\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+d\cdots\right.}}{2\sqrt{b^2-a^2}}\cdots\right.}{b}\cdots}{2(a^2-b^2)}\cdots\right.}{4b}\cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 266

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{2be\int\frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}\cdots}{2\sqrt{b^2-a^2}}\cdots\right.}{b}\cdots}{2(a^2-b^2)}\cdots\right.}{4b}\cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 756

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}\cdots}{2\sqrt{b^2-a^2}}\cdots\right.}{b}\cdots}{2(a^2-b^2)}\cdots\right.}{4b}\cdots$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 218

3.83. $\int\frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{}{\sqrt{e\sin}}\cdots}{}\right.}{b}}{2(a^2-b^2)}\cdots\right)$$

$$\overline{\qquad\qquad 4b \qquad\qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\downarrow$ 221

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\operatorname{arctanh}\cdots}{2\sqrt{b}e^{3}\cdots}\right.}{d}\right.}{b}}{2(a^2-b^2)}\cdots\right)$$

$$\overline{\qquad\qquad 4b \qquad\qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

3.83. $\int\frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

$\Big\downarrow$ 3042

$$e^2\left(\frac{\frac{(a^2+2b^2)\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}}{2\sqrt{b}e^3}\right.}{d}\right.}{b}}{2(a^2-b^2)}\right.$$

$$\overline{\qquad 4b \qquad}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3286

3.83. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c-}{\sqrt[4]{b^2-}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)}\right.}{}\right.}{b}}{2(a^2-b^2)}}{4b}\right.$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$e^2\left(\frac{\frac{\left(a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c-}{\sqrt[4]{b^2-}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)}\right.}{}\right.}{b}}{2(a^2-b^2)}}{4b}\right.$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

3.83. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

$\big\downarrow$ 3284

$$e^2\left(\frac{\frac{(a^2+2b^2)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}(b^2-a^2)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+da}}{d\sqrt{}}\right.}{b}}{2(a^2-b^2)}\right.}{4b}$$

$$\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

input
```
Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

output
```
(e*Sqrt[e*Sin[c + d*x]])/(2*b*d*(a + b*Cos[c + d*x])^2) + (e^2*(-((a*Sqrt[
e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))) + ((-2*a*Elliptic
F[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*d*Sqrt[e*Sin[c + d*x]]) +
((a^2 + 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2
+ b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt
[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2
))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2
]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Si
n[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqr
t[e*Sin[c + d*x]])))/b)/(2*(a^2 - b^2))))/(4*b)
```

3.83. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.83. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.83. $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2492 vs. $2(558) = 1116$.

Time = 7.71 (sec) , antiderivative size = 2493, normalized size of antiderivative = 4.67

method	result	size
default	Expression too large to display	2493

input
```
int((e*sin(d*x+c))^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(1/8*(e*sin(d*x+c))^(1/2)*e^2*(3*a^2*b^2*cos(d*x+c)^2-2*b^4*cos(d
*x+c)^2+a^4-2*a^2*b^2)/b^2/(a^2-b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2-1/6
4*(a^2+2*b^2)/b^2/(a^2-b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^
(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(
1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*e^2*a*(3/b^2*(-1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2
)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^
(1/2))+1/2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*
EllipticPi((1-sin(d*x+c))^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+(-7
*a^2+3*b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
/(-b^2*cos(d*x+c)^2+a^2)+1/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF(
(1-sin(d*x+c))^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/...
```

3.83.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

3.83.8 Giac [F]

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sin(d*x + c))^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{(e\,\sin(c+d\,x))^{3/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3, x)
```

3.84 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

3.84.1 Optimal result

Integrand size = 25, antiderivative size = 529

$$\begin{aligned}
&\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx \\
&= -\frac{(3a^2+2b^2)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}+\frac{(3a^2+2b^2)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d} \\
&\quad+\frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{a(3a^2+2b^2)\,e\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8b\,(a^2-b^2)^2\left(b+\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} \\
&\quad+\frac{5aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4\,(a^2-b^2)^2\,d\sqrt{\sin(c+dx)}} \\
&\quad-\frac{b(e\sin(c+dx))^{3/2}}{2\,(a^2-b^2)\,de(a+b\cos(c+dx))^2}-\frac{5ab(e\sin(c+dx))^{3/2}}{4\,(a^2-b^2)^2\,de(a+b\cos(c+dx))}
\end{aligned}$$

output
```
-1/2*b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2-5/4*a*b*(e*si
n(d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))-1/8*(3*a^2+2*b^2)*arctan(
b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^
(9/4)/d/b^(1/2)+1/8*(3*a^2+2*b^2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a
^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/4)/d/b^(1/2)-1/8*a*(3*a^2+2*b
^2)*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c
)^(1/2)/b/(a^2-b^2)^2/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-1/8*a*(3
*a^2+2*b^2)*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x
)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*s
in(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-
5/4*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/
sin(d*x+c)^(1/2)
```

3.84.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 12.24 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{\sqrt{e\sin(c+dx)}\left(-\frac{2b(7a^2-2b^2+5ab\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2}+\frac{\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(\frac{5a\sec(c+dx)\left(3\sqrt{2}a\left(a^2-b^2\right)^{3/4}\left(2\arctan\left(\cdots\right.\right.\right.}{\cdots}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
(Sqrt[e*Sin[c + d*x]]*((-2*b*(7*a^2 - 2*b^2 + 5*a*b*Cos[c + d*x])*Sin[c +
d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (Cos[c + d*x]*(a + b*Sqrt[C
os[c + d*x]^2])*((5*a*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTa
n[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2
- b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c +
d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
 + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(Sqrt[b]
*(-a^2 + b^2)) + (48*(4*a^2 + b^2)*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2,
 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(
3/2))/(3*(a^2 - b^2))))/Sqrt[Cos[c + d*x]^2]))/(12*(a - b)^2*(a + b)^2*(a
+ b*Cos[c + d*x])*Sqrt[Sin[c + d*x]])))/(8*d)
```

3.84.3 Rubi [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.90, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{(4a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx}{4(a^2-b^2)}-\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(4a-b\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4(a^2-b^2)}-\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3343

$$\frac{-\frac{\int -\frac{\left(8a^2+5b\cos(c+dx)a+2b^2\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{a^2-b^2}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(2\left(4a^2+b^2\right)+5ab\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2\left(4a^2+b^2\right)+5ab\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3346

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx+5a\int \sqrt{e\sin(c+dx)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+5a\int \sqrt{e\sin(c+dx)}dx}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3121

3.84. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}dx$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{5a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{5a\sqrt{e\sin(c+dx)}\int \sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{(3a^2+2b^2)\int \frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{10aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{2(a^2-b^2)}-\frac{5ab(e\sin(c+dx))^{3/2}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 3180$$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{be\int \frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}\cdots}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{2be\int \frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}d\cdots}{2b}\cdots}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}-\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \; 827$$

3.84. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{2be\left(\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}\cdots\right.}{2(a^2-b^2)}\cdots}{4(a^2-b^2)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\downarrow$ 218

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c\cdots}}\cdots}{\cdots}\right.}{2(a^2-b^2)}\cdots}{4(a^2-b^2)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\downarrow$ 221

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right.}{2b^{3/2}\sqrt{e}\cdots}\cdots\right.}{d}\cdots\right.}{2(a^2-b^2)}\cdots}{4(a^2-b^2)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

3.84. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\cdots\right.}{2b^{3/2}\sqrt{e}\cdots}\cdots\right.}{d}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3286

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\cdots\right.}{\cdots}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b\sqrt{e\sin(c+dx)}}-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\cdots\right.}{\cdots}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3284

3.84. $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

$$\frac{\frac{(3a^2+2b^2)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\,\text{E}\cdots}{bd\cdots}\cdots\right.}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b(e\sin(c+dx))^{3/2}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

input
```
Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

output
```
-1/2*(b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2) +
 ((-5*a*b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])) +
 ((10*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin
[c + d*x]]) + (3*a^2 + 2*b^2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*
x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(
Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^
(1/4)*Sqrt[e])))/d + (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi
/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin
[c + d*x]]) + (a*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*
x]])))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

3.84. $\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. 2(553) = 1106.

Time = 7.73 (sec) , antiderivative size = 2365, normalized size of antiderivative = 4.47

method	result	size
default	Expression too large to display	2365

input
```
int((e*sin(d*x+c))^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-1/8/(a^4-2*a^2*b^2+b^4)*(e*sin(d*x+c))^(3/2)*(-3*a^2*b^2*cos(d*
x+c)^2-2*b^4*cos(d*x+c)^2+7*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
^2-1/64*(3*a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/e^2/b^2/(e^2*(a^2-b^2)/b^2)^(1/4
)*2^(1/2)*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*
(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*e*a*(3/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-3/2/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Elli
pticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-9/8/(a^2-b^2)/b^2*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-
(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1
-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/2),1/(1-(-a^2+b^2)^(1/2)
/b),1/2*2^(1/2))-9/8/(a^2-b^2)/b^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2...
```

3.84.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**3,x)
```

output
```
Timed out
```

3.84.7 Maxima [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.84.8 Giac [F]

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\sin(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3,x)
```

output
```
int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3, x)
```

3.85 $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

3.85.1 Optimal result

Integrand size = 25, antiderivative size = 535

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx &= \frac{3\sqrt{b}(5a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}}+\frac{3\sqrt{b}(5a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{11/4}d\sqrt{e}}\\
&-\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{4\left(a^2-b^2\right)^2d\sqrt{e\sin(c+dx)}}\\
&+\frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&+\frac{3a(5a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^2\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}}\\
&-\frac{b\sqrt{e\sin(c+dx)}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2}-\frac{7ab\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))}\end{aligned}$$

output
```
3/8*(5*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(
1/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d/e^(1/2)+3/8*(5*a^2+2*b^2)*arctanh(b^(1/2
)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(11/4)
/d/e^(1/2)+7/4*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^
2)^2/d/(e*sin(d*x+c))^(1/2)-3/8*a*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a^2-b*(b-
(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/8*a*(5*a^2+2*b^2)*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/
d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/2*b*(e*sin(d*x+c))^(
1/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2-7/4*a*b*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)^2/d/e/(a+b*cos(d*x+c))
```

3.85.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 1226, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{\left(-\frac{b}{2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{7ab}{4(a^2-b^2)^2(a+b\cos(c+dx))}\right)\sin(c+dx)}{d\sqrt{e\sin(c+dx)}}$$
$$+ \frac{\sqrt{\sin(c+dx)}\left(-\frac{14ab\cos^2(c+dx)\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
((-1/2*b/((a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (7*a*b)/(4*(a^2 - b^2)^2*(
a + b*Cos[c + d*x])))*Sin[c + d*x])/(d*Sqrt[e*Sin[c + d*x]]) + (Sqrt[Sin[c
 + d*x]]*((-14*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2
*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Si
n[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) +
 (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(
a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 +
b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (2*(8*a^2 + 6*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8
 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqr
t[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq...
```

3.85.3 Rubi [A] (warning: unable to verify)

Time = 2.37 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.93, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3173}$$

$$-\frac{\int -\frac{4a-3b\cos(c+dx)}{2(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{b\sqrt{e\sin(c+dx)}}{2de(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ \mathbf{27}$$

$$\frac{\int \frac{4a-3b\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}dx}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ \mathbf{3042}$$

$$\frac{\int \frac{4a+3b\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ \mathbf{3343}$$

$$\frac{-\frac{\int -\frac{8a^2-7b\cos(c+dx)a+6b^2}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{a^2-b^2}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ \mathbf{27}$$

$$\frac{\frac{\int \frac{2\left(4a^2+3b^2\right)-7ab\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ \mathbf{3042}$$

$$\frac{\frac{\int \frac{2\left(4a^2+3b^2\right)+7ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ \mathbf{3346}$$

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx-7a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ \mathbf{3042}$$

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-7a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow \ \mathbf{3121}$$

3.85. $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{7a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{7a\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3120

$$\frac{\frac{3(5a^2+2b^2)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx-\frac{14a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}}{2(a^2-b^2)}-\frac{7ab\sqrt{e\sin(c+dx)}}{de(a^2-b^2)(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3181

$$\frac{\frac{3(5a^2+2b^2)\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}(b\sin(c+dx)}}{2\sqrt{b^2-a^2}}\right.}{2(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 266

$$\frac{\frac{3(5a^2+2b^2)\left(-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}\right.}{2(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 756

3.85. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

$$3(5a^2+2b^2)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int \frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}}{2\sqrt{b^2-a^2}}\right.$$

$$\overline{\qquad 2(a^2-b^2) \qquad}$$

$$\overline{\qquad 4\left(a^2-b^2\right) \qquad}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 218

$$3(5a^2+2b^2)\left(-\frac{2be\left(-\frac{\int \frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{}{\sqrt{e\sin(c}}}{}\right.$$

$$\overline{\qquad 2(a^2-b^2) \qquad}$$

$$\overline{\qquad 4\left(a^2-b^2\right) \qquad}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 221

$$3(5a^2+2b^2)\left(-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\right.}{2\sqrt{b}e^{3/2}}\right.}{d}\right.$$

$$\overline{\qquad 2(a^2-b^2) \qquad}$$

$$\overline{\qquad 4\left(a^2-b^2\right) \qquad}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

3.85. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

$$3\left(5a^2+2b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\cdots\right.}{2\sqrt{b}e^{3/2}\cdots}\right.}{d}\right.$$
$$2\left(a^2-b^2\right)$$
$$4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3286

$$3\left(5a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+\cdots}{\sqrt[4]{b^2-a\cdots}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3\cdots}}\right.}{\cdots}\right.$$
$$2\left(a^2-b^2\right)$$
$$4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3042

$$3\left(5a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+\cdots}{\sqrt[4]{b^2-a\cdots}}\right.}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3\cdots}}\right.}{\cdots}\right.$$
$$2\left(a^2-b^2\right)$$
$$4\left(a^2-b^2\right)$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

$\downarrow$ 3284

3.85. $\int\frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

$$\frac{3(5a^2+2b^2)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{b^2-a^2}},\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b^2-a^2}\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\cdots}{d\sqrt{b^2\cdots}}\cdots\right.}{2(a^2-b^2)} \cdots}{4\left(a^2-b^2\right)}$$

$$\frac{b\sqrt{e\sin(c+dx)}}{2de\left(a^2-b^2\right)\left(a+b\cos(c+dx)\right)^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

output
```
-1/2*(b*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2) + (
(-7*a*b*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])) + ((-
14*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c
+ d*x]]) + 3*(5*a^2 + 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c
+ d*x])/(-a^2 + b^2)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh
[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2
)^(3/4)*e^(3/2))))/d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi
/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2
])*d*Sqrt[e*Sin[c + d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (
c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2
 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

3.85. $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2219 vs. $2(559) = 1118$.

Time = 7.95 (sec) , antiderivative size = 2220, normalized size of antiderivative = 4.15

method	result	size
default	Expression too large to display	2220

input
```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2*b*e^3*(-1/8/(a^4-2*a^2*b^2+b^4)*(e*sin(d*x+c))^(1/2)*(-5*a^2*b^2*cos(d*
x+c)^2-2*b^4*cos(d*x+c)^2+9*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)
^2-3/64*(5*a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/e^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a
^2*e^2-b^2*e^2)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin
(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-
b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan
(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)*a*(3/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+3/4/a^2/(a^2-b^2)*(1-sin(d*x+c))^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ell
ipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-15/8/(a^2-b^2)/(-a^2+b^2)^(1/2)/b
*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)/(-a^2+b^2)^(1/2
)*b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c))^
(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+15/8/(a^2-b^2)/(-a^2+b^2)^(1/2
)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((1-sin(d*x+c...
```

3.85.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.85.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.85.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{e\sin(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*sqrt(e*sin(d*x + c))), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\sin\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x)
```

3.86 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

3.86.1 Optimal result

Integrand size = 25, antiderivative size = 611

$$
\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = & -\frac{5b^{3/2}(7a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} \\
& +\frac{5b^{3/2}(7a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} \\
& -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} \\
& -\frac{9ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} \\
& +\frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4\left(a^2-b^2\right)^3de\sqrt{e\sin(c+dx)}} \\
& -\frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
& -\frac{5ab(7a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(b+\sqrt{-a^2+b^2}\right)de\sqrt{e\sin(c+dx)}} \\
& -\frac{a(8a^2+37b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^3de^2\sqrt{\sin(c+dx)}}
\end{aligned}
$$

output
```
-5/8*b^(3/2)*(7*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^
(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(3/2)+5/8*b^(3/2)*(7*a^2+2*b^2)*arcta
nh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4
)/d/e^(3/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2)-9/
4*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+1/4*(5*b*(7*a^
2+2*b^2)-a*(8*a^2+37*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(1/2)
+5/8*a*b*(7*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c
))^(1/2)+5/8*a*b*(7*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^
(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b+(-a^2+b^2)^(1/2))/(e*s
in(d*x+c))^(1/2)+1/4*a*(8*a^2+37*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*
sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/e^2/sin(d*x+c)^(1/2)
```

3.86.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.89 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \frac{\sin^2(c+dx)\left(-\frac{2(-3a^2b-b^3+a^3\cos(c+dx)+3ab^2\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^3}+\cdots\right.}{d(e\sin(c+dx))^{3/\cdots}}$$

$$-\frac{\sin^{\frac{3}{2}}(c+dx)\left(\frac{(8a^3b+37ab^3)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-\cdots}\right.\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
(Sin[c + d*x]^2*((-2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c +
d*x])*Csc[c + d*x])/(a^2 - b^2)^3 + (b^3*Sin[c + d*x])/(2*(a^2 - b^2)^2*(a
 + b*Cos[c + d*x])^2) + (13*a*b^3*Sin[c + d*x])/(4*(a^2 - b^2)^3*(a + b*Co
s[c + d*x]))))/(d*(e*Sin[c + d*x])^(3/2)) - (Sin[c + d*x]^(3/2)*(((8*a^3*b
 + 37*a*b^3)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] +
 Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 -
Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c
 + d*x]^2)) + (2*(8*a^4 + 72*a^2*b^2 + 10*b^4)*Cos[c + d*x]*(((1/8 + I/8)*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c +...
```

3.86.3 Rubi [A] (warning: unable to verify)

Time = 2.91 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.94, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{4a-5b\cos(c+dx)}{2(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a-5b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}dx}{4(a^2-b^2)}-\frac{b}{2de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a+5b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4(a^2-b^2)}-\frac{b}{2de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3343}$$

$$\frac{-\frac{\int -\frac{8a^2-27b\cos(c+dx)a+10b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{a^2-b^2}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b}{2de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{2\left(4a^2+5b^2\right)-27ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b}{2de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{2\left(4a^2+5b^2\right)+27ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))}}{4(a^2-b^2)}-\frac{b}{2de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{2\int \frac{\left(8a^4+72b^2a^2+b\left(8a^2+37b^2\right)\cos(c+dx)a+10b^4\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+d}}{4(a^2-b^2)}-\frac{b}{2de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

3.86. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int\frac{\left(2\left(4a^4+36b^2a^2+5b^4\right)+ab\left(8a^2+37b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\int\frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2\left(4a^4+36b^2a^2+5b^4\right)+ab\left(8a^2+37b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3346}$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{a\left(8a^2+37b^2\right)\int\sqrt{e\sin(c+dx)}dx+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(c+d}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{a\left(8a^2+37b^2\right)\int\sqrt{e\sin(c+dx)}dx+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+dx)}(a+b\cos(}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\frac{a\left(8a^2+37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+d}}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

3.86. $\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}dx$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{\frac{a\left(8a^2+37b^2\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}+5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{9ab}{de(a^2-b^2)\sqrt{e\sin(c+d\cdots}}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3119$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx+\frac{2a\left(8a^2+37b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{\cdots}{de(a^2-b^2)\sqrt{e\sin(\cdots}}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3180$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}\cdots\right.}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}\cdots}{4\left(a^2-b^2\cdots\right.}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}d\cdots}{2b}\right.}{e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}\cdots}{4\left(a^2-b\cdots\right.}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 827$$

3.86. $\displaystyle\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}\cdots\right.}{\cdots}}{2(a^2-b^2)\cdots}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 218

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2b}\right)}{d}-\frac{ae\int\frac{\cdots}{\sqrt{e\sin(c\cdots}}}{\cdots}\cdots\right.}{\cdots}}{2(a^2-b^2)\cdots}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 221

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots\right.}{e^2\cdots}}{2(a^2-b^2)\cdots}$$

$4\,(\cdots$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

3.86. $\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^2}\right)}dx}{2b}-\cdots\right)}{e^2(\cdots}}{2(a^2-b^2)} \cdots}{4(\cdots}$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3286

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+d\cdots\right.}}{2b\sqrt{e\sin(c+dx)}}\cdots\right)}{\cdots}}{2(a^2-b^2)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(b\sin(c+d\cdots\right.}}{2b\sqrt{e\sin(c+dx)}}\cdots\right)}{\cdots}}{2(a^2-b^2)}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

3.86. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{3284}$$

$$\frac{\frac{2\left(5b\left(7a^2+2b^2\right)-a\left(8a^2+37b^2\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{5b^2\left(7a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{d}+\frac{ae\sqrt{\sin(c+dx)}\,\text{EllipticPi}\left(\cdots\right.}{bd\left(b-\sqrt{b^2-\cdots}\right.}\cdots\right.}{\cdots}}{2(a^2-b^2)\cdots}\cdots$$

$$\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}(a+b\cos(c+dx))^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

output
```
-1/2*b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]) + ((-
9*a*b)/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + ((2*(
5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*Cos[c + d*x]))/((a^2 - b^2)*d*e*S
qrt[e*Sin[c + d*x]]) - ((2*a*(8*a^2 + 37*b^2)*EllipticE[(c - Pi/2 + d*x)/2
, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + 5*b^2*(7*a^2 + 2*b^2)*
((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*b^(
3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/
(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*Elli
pticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*EllipticPi[
(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(
b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 - b^2)*e^2))/(2*(
a^2 - b^2)))/(4*(a^2 - b^2))
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

3.86. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.86. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.86.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2836 vs. 2(631) = 1262.

Time = 10.14 (sec) , antiderivative size = 2837, normalized size of antiderivative = 4.64

method	result	size
default	Expression too large to display	2837

input
```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(b^2/e^4/(a-b)^3/(a+b)^3*(1/8*(e*sin(d*x+c))^(3/2)*e^2*(-11*a^2*b
^2*cos(d*x+c)^2-2*b^4*cos(d*x+c)^2+15*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^
2+a^2*e^2)^2+1/8*(35/8*a^2+5/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*
(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(
e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d
*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a^
2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)/
b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-(-3*a^2-b^2)/e^4/(a^2-b^2)^3/(e*sin(d
*x+c))^(1/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e*a*((-a^2-3*b^2)/(a^2-b^2
)^3*(2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos
(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)+4*a^2*b^2/(a-b)/(a+b)*(1/4*b^
2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d
*x+c)^2+a^2)^2+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)-11/16/a^2/(a^2-b^2)^2*
(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+3/8/a^4/(
a^2-b^2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((1-sin(d*x+c))^(1/2),1/2*2^(1...
```

3.86.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.86.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.86.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3\,(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(3/2)), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x)
```

3.87 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

3.87.1 Optimal result

Integrand size = 25, antiderivative size = 629

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx &= \frac{7b^{5/2}(9a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}} \\
&+\frac{7b^{5/2}(9a^2+2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}} \\
&-\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} \\
&-\frac{11ab}{4\left(a^2-b^2\right)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} \\
&+\frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12\left(a^2-b^2\right)^3de(e\sin(c+dx))^{3/2}} \\
&+\frac{a(8a^2+69b^2)\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{12\left(a^2-b^2\right)^3de^2\sqrt{e\sin(c+dx)}} \\
&-\frac{7ab^2(9a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}} \\
&-\frac{7ab^2(9a^2+2b^2)\,\text{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8\left(a^2-b^2\right)^3\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right)de^2\sqrt{e\sin(c+dx)}}\end{aligned}$$

output
```
7/8*b^(5/2)*(9*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/e^(5/2)+7/8*b^(5/2)*(9*a^2+2*b^2)*arctan
h(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(15/4)
/d/e^(5/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2)-11/
4*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/12*(7*b*(9*a
^2+2*b^2)-a*(8*a^2+69*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(3/2
)-1/12*a*(8*a^2+69*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/
(a^2-b^2)^3/d/e^2/(e*sin(d*x+c))^(1/2)+7/8*a*b^2*(9*a^2+2*b^2)*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/
4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)
^3/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+7/8*a*b^2*(9*a^
2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*
x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(
1/2)
```

3.87.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.26 (sec) , antiderivative size = 1308, normalized size of antiderivative = 2.08

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
((b^3/(2*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (15*a*b^3)/(4*(a^2 - b^2)
^3*(a + b*Cos[c + d*x])) - (2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2
*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^3))*Sin[c + d*x]^3)/(d*(e*Si
n[c + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(8*a^3*b + 69*a*b^3)*Cos[c +
d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*S
qrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c +
d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt
[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1
/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b
^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))
))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(8*a^4 - 120*a^2*b^2 -
 42*b^4)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt
[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)...
```

3.87.3 Rubi [A] (warning: unable to verify)

Time = 2.99 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.95, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3120, 3181, 266, 756, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3173}$$

$$-\frac{\int -\frac{4a-7b\cos(c+dx)}{2(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4a-7b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{4a+7b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3343$$

$$\frac{-\frac{\int -\frac{8a^2-55b\cos(c+dx)a+14b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{a^2-b^2}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{2\left(4a^2+7b^2\right)-55ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}dx}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{2\left(4a^2+7b^2\right)+55ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 3345$$

$$\frac{\frac{\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{2\int -\frac{8a^4-120b^2a^2+b\left(8a^2+69b^2\right)\cos(c+dx)a-42b^4}{2(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 27$$

3.87. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{\int \frac{2\left(4a^4-60b^2a^2-21b^4\right)+ab\left(8a^2+69b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{2\left(4a^4-60b^2a^2-21b^4\right)-ab\left(8a^2+69b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3346

$$\frac{\frac{\frac{a\left(8a^2+69b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21b^2\left(9a^2+2b^2\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{a\left(8a^2+69b^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}dx-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3121

$$\frac{\frac{\frac{\frac{a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{11ab}{de(a^2-b^2)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

3.87. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{\frac{a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx)\right)}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}-\frac{\ldots}{de(\ldots}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3120

$$\frac{\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\int \frac{1}{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{3e^2\left(a^2-b^2\right)}+\frac{2\left(7b\left(9a^2+2b^2\right)-a\left(8a^2+69b^2\right)\cos(c+dx\ldots\right.}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}}{2(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 3181

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{be\int \frac{1}{\sqrt{e\sin(c+dx)}\left(b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2\right)}d(e\sin(c+dx))}{d}-\frac{a\int \frac{\ldots}{\sqrt{e\sin(c+dx)}(\ldots}}{2\ldots}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}}{4\ldots}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 266

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{2be\int \frac{1}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin\ldots\right.}}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}}{4\left(a^2-\ldots\right.}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\big\downarrow$ 756

3.87. $\displaystyle\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{}}{2e\sqrt{b^2-a^2}}\right.}{d}\right.$$
$$3e^2\left(a^2-b^2\right)$$
$$2(a^2-b^{\cdots}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$
$$\Big\downarrow \text{218}$$

$$\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+dx)}}{2e\sqrt{b^2-a^2}}-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}-\cdots\right.$$
$$3e^2\left(a^2-b^2\right)$$
$$2(a^2-b^2)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$
$$\Big\downarrow \text{221}$$

$$\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a^{\cdots}}\right.}}{2\sqrt{b^2-a^2}}\right.$$
$$3e^2\left(a^2-b^2\right)$$
$$2(a^2-b^2)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$
$$\Big\downarrow \text{3042}$$

3.87. $\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}}-\frac{a\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+dx)+\sqrt{b^2-a}\right.}}{2\sqrt{b^2-a^2}}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3286

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}(}}{2\sqrt{b^2-a^2}\sqrt{e}}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2\sqrt{b^2-a^2}\sqrt{e\sin(c+dx)}}-\frac{a\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}(}}{2\sqrt{b^2-a^2}\sqrt{e}}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}$$

3.87. $\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

$\downarrow$ 3284

$$\frac{\frac{\frac{2a\left(8a^2+69b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}-21b^2\left(9a^2+2b^2\right)\left(-\frac{2be\left(-\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}e^{3/2}\left(b^2-a^2\right)^{3/4}}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}}{d\sqrt{b}}\right.}{3e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{3/2}\left(a+b\cos(c+dx)\right)^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

output
```
-1/2*b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)) + (
(-11*a*b)/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) +
((2*(7*b*(9*a^2 + 2*b^2) - a*(8*a^2 + 69*b^2)*Cos[c + d*x]))/(3*(a^2 - b^2
)*d*e*(e*Sin[c + d*x])^(3/2)) + ((2*a*(8*a^2 + 69*b^2)*EllipticF[(c - Pi/2
 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) - 21*b^2*(9*a^2
 + 2*b^2)*((-2*b*e*(-1/2*ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2
)^(1/4)]/(Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2)) - ArcTanh[(Sqrt[b]*Sqrt[e]*S
in[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(-a^2 + b^2)^(3/4)*e^(3/2))))/
d + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqr
t[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(Sqrt[-a^2 + b^2]*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*S
in[c + d*x]])))/(3*(a^2 - b^2)*e^2))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

3.87. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

rule 3181
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[-a/(2*q)   Int[1/(
Sqrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Simp[b*(g/f)   Subst[
Int[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - S
imp[a/(2*q)   Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

3.87. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.87.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2680 vs. $2(649) = 1298$.

Time = 11.22 (sec) , antiderivative size = 2681, normalized size of antiderivative = 4.26

method	result	size
default	Expression too large to display	2681

input
```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(b^2/e^4/(a-b)^3/(a+b)^3*(1/8*(e*sin(d*x+c))^(1/2)*e^2*(-13*a^2*b
^2*cos(d*x+c)^2-2*b^4*cos(d*x+c)^2+17*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e^
2+a^2*e^2)^2+7/64*(9*a^2+2*b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2
)*2^(1/2)*(ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*
(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/3*(-3*a^2-b^2)/e^4/(a^2-b
^2)^3/(e*sin(d*x+c))^(3/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*a*(1/3*(
-a^2-3*b^2)/(a^2-b^2)^3/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)
*((1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((
1-sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+4*a^2*b^2/(a+b
)/(a-b)*(1/4*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*c
os(d*x+c)^2+a^2)^2+1/16*b^2*(13*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2+a^2)+13/32/a^2/(a^2-b^2)^2*(1-sin(
d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^
2)^2*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))*b^2-
45/64/(a^2-b^2)^2/(-a^2+b^2)^(1/2)/b*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)...
```

3.87.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.87.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.87.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(5/2)), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\,\sin(c+d\,x))^{5/2}\,(a+b\,\cos(c+d\,x))^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x)
```

3.88 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

3.88.1 Optimal result

Integrand size = 25, antiderivative size = 700

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx &= -\frac{9b^{7/2}(11a^2+2b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8(-a^2+b^2)^{17/4}de^{7/2}} \\
&+\frac{9b^{7/2}(11a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8(-a^2+b^2)^{17/4}de^{7/2}} \\
&-\frac{b}{2(a^2-b^2)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} \\
&-\frac{13ab}{4(a^2-b^2)^2de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} \\
&+\frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20(a^2-b^2)^3de(e\sin(c+dx))^{5/2}} \\
&-\frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20(a^2-b^2)^4de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{9ab^3(11a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b-\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^4\left(b-\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&+\frac{9ab^3(11a^2+2b^2)\operatorname{EllipticPi}\left(\frac{2b}{b+\sqrt{-a^2+b^2}},\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)\sqrt{\sin(c+dx)}}{8(a^2-b^2)^4\left(b+\sqrt{-a^2+b^2}\right)de^3\sqrt{e\sin(c+dx)}} \\
&-\frac{3a(8a^4-64a^2b^2-139b^4)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20(a^2-b^2)^4de^4\sqrt{\sin(c+dx)}}
\end{aligned}$$

output
```
-9/8*b^(7/2)*(11*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)
^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(7/2)+9/8*b^(7/2)*(11*a^2+2*b^2)*arc
tanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17
/4)/d/e^(7/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2)-
13/4*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/20*(9*b*(
11*a^2+2*b^2)-a*(8*a^2+109*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))
^(5/2)-3/20*(15*b^3*(11*a^2+2*b^2)+a*(8*a^4-64*a^2*b^2-139*b^4)*cos(d*x+c)
)/(a^2-b^2)^4/d/e^3/(e*sin(d*x+c))^(1/2)-9/8*a*b^3*(11*a^2+2*b^2)*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c
+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b
^2)^4/d/e^3/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-9/8*a*b^3*(11*a^2+2*
b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)
^(1/2)/(a^2-b^2)^4/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/20*a*
(8*a^4-64*a^2*b^2-139*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))
^(1/2)/(a^2-b^2)^4/d/e^4/sin(d*x+c)^(1/2)
```

3.88.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.02 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \frac{\sin^4(c+dx)\left(-\frac{2(50a^2b^3+10b^5+3a^5\cos(c+dx)-24a^3b^2\cos(c+dx)-39ab^4\cos}{5(a^2-b^2)^4}\right.}{}$$

$$-\frac{3\sin^{\frac{7}{2}}(c+dx)\left(\frac{(8a^5b-64a^3b^3-139ab^5)\cos^2(c+dx)\left(3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
(Sin[c + d*x]^4*((-2*(50*a^2*b^3 + 10*b^5 + 3*a^5*Cos[c + d*x] - 24*a^3*b^
2*Cos[c + d*x] - 39*a*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^4) -
(2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c + d*x])*Csc[c + d*x]
^3)/(5*(a^2 - b^2)^3) - (b^5*Sin[c + d*x])/(2*(a^2 - b^2)^3*(a + b*Cos[c +
 d*x])^2) - (21*a*b^5*Sin[c + d*x])/(4*(a^2 - b^2)^4*(a + b*Cos[c + d*x]))
))/(d*(e*Sin[c + d*x])^(7/2)) - (3*Sin[c + d*x]^(7/2)*(((8*a^5*b - 64*a^3*
b^3 - 139*a*b^5)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 +
(Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b
^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x
]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c +
d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d
*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[
1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - S
in[c + d*x]^2)) + (2*(8*a^6 - 64*a^4*b^2 - 304*a^2*b^4 - 30*b^6)*Cos[c + d
*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2
 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[...
```

3.88.3 Rubi [A] (warning: unable to verify)

Time = 3.65 (sec) , antiderivative size = 679, normalized size of antiderivative = 0.97, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 3173, 27, 3042, 3343, 27, 3042, 3345, 27, 3042, 3345, 27, 3042, 3346, 3042, 3121, 3042, 3119, 3180, 266, 827, 218, 221, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3173}$$

$$-\frac{\int -\frac{4a-9b\cos(c+dx)}{2(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}dx}{2\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\int \frac{4a-9b\cos(c+dx)}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{\int \frac{4a+9b\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{3343}$$

$$\frac{-\frac{\int -\frac{8a^2-91b\cos(c+dx)a+18b^2}{2(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{a^2-b^2}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{\int \frac{2\left(4a^2+9b^2\right)-91ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}dx}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{\frac{\int \frac{2\left(4a^2+9b^2\right)+91ab\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{7/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{3345}$$

$$\frac{\frac{\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}-\frac{2\int -\frac{3\left(2\left(4a^4-28b^2a^2-15b^4\right)+ab\left(8a^2+109b^2\right)\cos(c+dx)\right)}{2(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+d}}{4\left(a^2-b^2\right)}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

3.88. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

$$\frac{\frac{\frac{3\int\frac{2\left(4a^4-28b^2a^2-15b^4\right)+ab\left(8a^2+109b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}dx}{5e^2\left(a^2-b^2\right)}+\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}-$$
$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\int\frac{2\left(4a^4-28b^2a^2-15b^4\right)-ab\left(8a^2+109b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)}{\left(e\cos\left(c+dx-\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{5e^2\left(a^2-b^2\right)}+\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}}{2(a^2-b^2)}-\frac{13ab}{de(a^2-b^2)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}}{4\left(a^2-b^2\right)}$$
$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3345

$$\frac{\frac{3\left(-\frac{2\int\frac{\left(8a^6-64b^2a^4-304b^4a^2+b\left(8a^4-64b^2a^2-139b^4\right)\cos(c+dx)a-30b^6\right)\sqrt{e\sin(c+dx)}}{2(a+b\cos(c+dx))}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2\left(a^2-b^2\right)}+\frac{2(9b}{}}{2(a^2-b^2)}}{4\left(a^2-b^2\right)}$$
$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{3\left(-\frac{\int\frac{\left(2\left(4a^6-32b^2a^4-152b^4a^2-15b^6\right)+ab\left(8a^4-64b^2a^2-139b^4\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)}{5e^2\left(a^2-b^2\right)}+\frac{2(}{}}{2(a^2-b^2)}}{4\left(a^2-b^2\right)}$$
$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$\downarrow$ 3042

3.88. $\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\int \frac{\sqrt{-e\cos\left(c+dx+\frac{\pi}{2}\right)}\left(2\left(4a^6-32b^2a^4-152b^4a^2-15b^6\right)+ab\left(8a^4-64b^2a^2-139b^4\right)\sin\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+\right.}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right.$$

$$\overline{5e^2\left(a^2-b^2\right)}$$

$$\overline{2(a^2-b^2)}$$

$$\overline{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3346}$$

$$3\left(-\frac{a\left(8a^4-64a^2b^2-139b^4\right)\int\sqrt{e\sin(c+dx)}dx-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)+\frac{2\left(9b\left(11a^2+\right.\right.}{5de}$$

$$\overline{5e^2\left(a^2-b^2\right)}$$

$$\overline{2(a^2-b^2)}$$

$$\overline{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$3\left(-\frac{a\left(8a^4-64a^2b^2-139b^4\right)\int\sqrt{e\sin(c+dx)}dx-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right)+\frac{2\left(9b\left(1\right.\right.}{}$$

$$\overline{5e^2\left(a^2-b^2\right)}$$

$$\overline{2(a^2-b^2)}$$

$$\overline{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3121}$$

$$3\left(-\frac{\frac{a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right.$$

$$\overline{5e^2\left(a^2-b^2\right)}$$

$$\overline{2(a^2-b^2)}$$

$$\overline{4\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

3.88. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\frac{a\left(8a^4-64a^2b^2-139b^4\right)\sqrt{e\sin(c+dx)}\int\sqrt{\sin(c+dx)}dx}{\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+dx)\right)}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right.$$

$$5e^2\left(a^2-b^2\right)$$

$$2(a^2-b^2)$$

$$4\left(a^2-b^2\right)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\downarrow 3119$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\int\frac{\sqrt{e\cos\left(c+dx-\frac{\pi}{2}\right)}}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{e^2\left(a^2-b^2\right)}-\frac{2\left(15b^3\left(11a^2+2b^2\right)+a\left(8a^4-64a^2b^2-139b^4\right)\cos(c+\right.}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}\right.$$

$$5e^2\left(a^2-b^2\right)$$

$$2(a^2-b^2)$$

$$4\left(a^2-b^2\right)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\downarrow 3180$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{be\int\frac{\sqrt{e\sin(c+dx)}}{b^2\sin^2(c+dx)e^2+\left(a^2-b^2\right)e^2}d(e\sin(c+dx))}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2\right.}}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$5e^2\left(a^2-b^2\right)$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\downarrow 266$$

3.88. $\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}dx$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\int\frac{e^2\sin^2(c+dx)}{b^2e^4\sin^4(c+dx)+\left(a^2-b^2\right)e^2}d\sqrt{e\sin(c+dx)}}{d}-\frac{ae\int\frac{}{\sqrt{e\sin(c+dx)}\left(\sqrt{\cdots}\right.}}{}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\overline{\qquad 5e^2\left(a^2-b^2\right)}$$

$$\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow 827$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\left(\frac{\int\frac{1}{be^2\sin^2(c+dx)+\sqrt{b^2-a^2}e}d\sqrt{e\sin(c+dx)}}{2b}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(\cdots}}{2b}\right.}{d}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)\left(e\sin(c+dx)\right)^{5/2}\left(a+b\cos(c+dx)\right)^2}$$

$$\Big\downarrow 218$$

3.88. $\int\frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+dx)}{\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-a^2}}-\frac{\int\frac{1}{\sqrt{b^2-a^2}e-be^2\sin^2(c+dx)}d\sqrt{e\sin(c+\ldots}}{2b}\ldots\right)}{d}\ldots\right.}{e^2\left(a^2-b^2\right)\ldots}\ldots\right)$$

$$\overline{5e^2(\ldots}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \ 221$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+\ldots\right.}}{2b}\ldots\right.}{e^2\left(a^2-b^2\right)\ldots}\ldots\right)$$

$$\overline{5e^2\left(a^2-b^{\ldots}\right.}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

3.88. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \text{3042}$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b}+\frac{ae\int\frac{1}{\sqrt{e\sin(c+dx)}\left(b\sin(c+\right.}}{2b}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$5e^2\left(a^2-b^2\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3286}$$

3.88. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int}{}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{ae\sqrt{\sin(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx)}\left(\sqrt{b^2-a^2}-b\sin(c+dx)\right)}dx}{2b\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\int}{}\right.}{e^2\left(a^2-b^2\right)}\right.$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

3.88. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \; 3284$$

$$\frac{\frac{2\left(9b\left(11a^2+2b^2\right)-a\left(8a^2+109b^2\right)\cos(c+dx)\right)}{5de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}}+\frac{3\left(-\frac{\frac{2a\left(8a^4-64a^2b^2-139b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}-15b^4\left(11a^2+2b^2\right)\left(-\frac{2be\left(\frac{\arctan\left(\frac{\sqrt{b}\sqrt{e}\sin(c+\cdots}{\sqrt[4]{b^2-\cdots}}\right)}{2b^{3/2}\sqrt{e}\sqrt[4]{b^2-\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots}{\cdots}$$

$$\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}$$

input
```
Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]
```

output
```
-1/2*b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)) + (
(-13*a*b)/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)) +
((2*(9*b*(11*a^2 + 2*b^2) - a*(8*a^2 + 109*b^2)*Cos[c + d*x]))/(5*(a^2 - b
^2)*d*e*(e*Sin[c + d*x])^(5/2)) + (3*((-2*(15*b^3*(11*a^2 + 2*b^2) + a*(8*
a^4 - 64*a^2*b^2 - 139*b^4)*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c +
 d*x]]) - ((2*a*(8*a^4 - 64*a^2*b^2 - 139*b^4)*EllipticE[(c - Pi/2 + d*x)/
2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) - 15*b^4*(11*a^2 + 2*b^
2)*((-2*b*e*(ArcTan[(Sqrt[b]*Sqrt[e]*Sin[c + d*x])/(-a^2 + b^2)^(1/4)]/(2*
b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e]) - ArcTanh[(Sqrt[b]*Sqrt[e]*Sin[c + d*x
])/(-a^2 + b^2)^(1/4)]/(2*b^(3/2)*(-a^2 + b^2)^(1/4)*Sqrt[e])))/d + (a*e*E
llipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Elliptic
Pi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])))/((a^2 - b^2)*e^2)))/
(5*(a^2 - b^2)*e^2))/(2*(a^2 - b^2)))/(4*(a^2 - b^2))
```

3.88. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3180
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Simp[a*(g/(2*b))   Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Simp[a*(g/(2*b))   In
t[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Simp[b*(g/f)   Su
bst[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /
; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.88. $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 3346
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int
[(g*Cos[e + f*x])^p, x], x] + Simp[(b*c - a*d)/b   Int[(g*Cos[e + f*x])^p/(
a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 -
 b^2, 0]
```

3.88.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3078 vs. $2(716) = 1432$.

Time = 12.10 (sec) , antiderivative size = 3079, normalized size of antiderivative = 4.40

method	result	size
default	Expression too large to display	3079

input
```
int(1/(a+cos(d*x+c)*b)^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2*e^3*b*(-b^4/e^6/(a+b)^4/(a-b)^4*(1/8*(e*sin(d*x+c))^(3/2)*e^2*(-19*a^2*
b^2*cos(d*x+c)^2-2*b^4*cos(d*x+c)^2+23*a^4-2*a^2*b^2)/(-b^2*cos(d*x+c)^2*e
^2+a^2*e^2)^2+1/8*(99/8*a^2+9/4*b^2)/b^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)
*(ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+
(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2*arctan(2^(1/2)/(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+2*arctan(2^(1/2)/(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)))-1/5*(-3*a^2-b^2)/e^4/(a-b)^3/(a+b)^3
/(e*sin(d*x+c))^(5/2)-2*b^2*(5*a^2+b^2)/e^6/(a+b)^4/(a-b)^4/(e*sin(d*x+c))
^(1/2))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^3*a*(-1/5*(-a^2-3*b^2)/(a^2-b^
2)^3/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/sin(d*x+c)/(cos(d*x+c)^2-1)*(6*(1-s
in(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((1-sin(
d*x+c))^(1/2),1/2*2^(1/2))-3*(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(7/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))+6*sin(d*x+c)*c
os(d*x+c)^4-8*cos(d*x+c)^2*sin(d*x+c))+6*b^2*(a^2+b^2)/(a^2-b^2)^4*(2*(1-s
in(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((1-sin(
d*x+c))^(1/2),1/2*2^(1/2))-(1-sin(d*x+c))^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((1-sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)-4*a^2*b^4/(a-b)^2/(a+b)^2*(1/4*b^2/e/a^2
/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c...
```

3.88.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.88.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.88.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c)+a)^3 (e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c) + a)^3*(e*sin(d*x + c))^(7/2)), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),x)
```

output
```
int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```